In the Footsteps of the Real

The Indiciary Paradigm in the Genesis of Quantum Theory

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The Indiciary Paradigm in the Genesis of Quantum Theory

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Abstract

This article offers a philosophical micro-history of the genesis of quantum theory (1900–1930) through what is termed an *indiciary–abductive* paradigm. Against standard narratives centred on axioms, formalism or "revolutions," the paper reconstructs early quantum episodes as sequences of clue-driven inferences, where small phenomenological mismatches and structural regularities acquire normative weight disproportionate to their empirical modesty. Drawing on Ginzburg's indiciary paradigm, Peircean abduction and recent work on epistemic aesthetics, the article develops a formal decision rule in which candidate hypotheses are evaluated by a composite functional of indicial weight, aesthetic value (parsimony, symmetry, coherence of limits, unification), programme fecundity and correspondence constraints. This framework is then applied micro-historically to canonical cases in the quantum germ—Planck, Einstein, Bohr, de Broglie, Heisenberg, Schrödinger and Pauli—each analysed as a transition from a specific clue set and background model to an abductively introduced hypothesis that opens a research programme. The reconstruction shows that quantum theory emerged not from an abrupt repudiation of classical rationality but from a disciplined practice of reading traces, in which aisthesis (trained sensitivity to experimental and conceptual detail) and epistēmē (formal, aesthetic and statistical criteria of theory choice) are tightly coupled. The conclusion generalises this indicial logic as a model of scientific discovery under weak, heterogeneous evidence, with implications for model selection, the role of aesthetic criteria in physics, and the design of comparative micro-histories of other scientific "revolutions."

Keywords: indiciary paradigm; abduction; quantum theory; epistemic aesthetics; micro-history of science; model selection; philosophy of scientific Discovery.

1. Introduction

The gestation period of quantum theory (1900–1930) is best understood, against the standard account—focused on experimental techniques, formal models and axioms—if one superimposes upon it a grammar of discovery woven from three epistemic strands: (i) the reading of clues (discrete traces, phenomenological discontinuities and "minor details" in the laboratory) as signs of latent structures (Ginzburg, 1992); (ii) Peircean abduction as a rule for moving from the surprising to the plausible (Peirce, 1998; Aliseda, 2006); and (iii) an epistemic aesthetics which, far from being a mere subjective preference, operates as a normative filter—simplicity, symmetry, coherence of limits—for prioritising conjectures under incomplete evidence (Ivanova & French, 2020; McAllister, 1996). The result is a philosophical history of science that articulates rationality and sensibility: *aisthesis* (perception, pattern, rhythm, gesture) and *epistēmē* (reason, communication, rule, proof)

converge in a single cognitive resource. We shall call this epistemic convergence the *indiciary paradigm*, in complementary and critical—rather than antagonistic—contrast to the Galilean–Newtonian paradigm that dominated scientific modernity.

It is worth clarifying that the use of *aisthesis* in this paper does not follow the Platonic restriction that confines it to the domain of mere sensible *doxa*, but the broader sense preserved in pre-Platonic and Aristotelian traditions: the field of conscious experience in which sensation, judgement, emotion and belief are interwoven. In the reading of Protagoras reconstructed by Rojas Parma, *aisthesis* "covers a broad range of meanings, including sensation, awareness of external objects or facts, feelings, emotions, etc." (Rojas Parma, 2015), so that the thesis *aisthesis is epistēmē* does not glorify subjectivism, but rather inscribes knowing within lived experience, traversed by criteria of evidence and security. In the opposite direction to the Platonic reductionism of the *Theaetetus*, the Aristotelian reworking of perception in *De Anima*—where sensible and intellectual faculties function as gears of the same cognitive device—allows *aisthesis* to be construed as the operative threshold of intellection.

When we speak, therefore, of a philosophical history of science in which *aisthesis* and *epistēmē* converge, we are not proposing an arbitrary aestheticisation of knowledge, but the technical reactivation of that tradition in which qualified perception and theoretical judgement form a single economy of access to the real.

Within this theoretical–methodological horizon, the reading of clues proposed here can be described as a scientific update of *kairos* as semiotic opportunity: each experimental trace functions as a sign of opportune occasion whose correct interpretation requires a specific synchrony between *representamen*, object and interpretant. As Ramírez Suárez has shown, genuine Peircean semiosis depends upon "the attunement and affinity or compatibility between ontological, epistemological and semiotic categorical coordinates" at a precise moment, such that "if it occurs earlier, it degenerates by anticipation; if it occurs later, it degenerates by delay. It is the 'now' of the actualisation of possibility and real reality. It is the semiotic *kairos*" (Ramírez Suárez, 2007). The Peircean *musement* then operates as an abductive praxis through which the physicist explores, in a relatively conscious way, the constellation of possible interpretants until, at a certain point, some particular clue, abductively detected, acquires the status of sign of opportune occasion and triggers a conjecture that is a candidate for formalisation.

The central thesis of this paper can be expressed operationally as follows: the acceptability of early quantum conjectures is better explained if we model theoretical practice as an optimisation under constraints. Given a set of clues $S$and a classical background K, the preferred hypothesis H^*maximises an epistemic utility

$$H^* \in \arg\max_{H \in \mathcal{H}} (W(S; H, K))\, \mathcal{A}(H)\, \Phi(H; K) \text{subject to} \mathcal{L}(H) \leq \varepsilon,$$

where $W$is an indicial weight (likelihood ratios penalised by complexity), $\mathcal{A}$an aesthetic functional (simplicity, symmetry, coherence of limits), Φfecundity (unification and prediction), and $\mathcal{L}$a measure of correspondence with entrenched regimes (e.g. $\hbar \to 0$) (Royall, 1997; Akaike, 1974; Schwarz, 1978; Rissanen, 1978; Ivanova & French, 2020; McAllister, 1996). We shall call this ampliative, non-monotonic epistemological procedure for discovery—one that, starting from singular traces Sevaluated against a background K, selects conjectures Hvia the decision rule just given, integrating indicial weight W, operative aesthetic filter $\mathcal{A}$, fecundity $\Phi$and correspondence constraint $\mathcal{L}$(Ginzburg, 1992; Peirce, 1998; Aliseda, 2006)—an *indiciary-abductive logic*.

On this basis, the aims of the present article are: (1) to reconstruct the indiciary paradigm as a legitimate epistemic procedure and distinguish it from the Galilean–Newtonian one; (2) to make abduction (Peirce) explicit as a logical–operative mechanism

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in the genesis of hypotheses; and (3) to operationalise epistemic aesthetics—via an $\mathcal{A}$with components $\mathcal{C}$, Sym, $\mathcal{L}$, $\mathcal{U}$—and apply it to quantum episodes. The quality of Hincludes both the integrative likelihood of heterogeneous clues and aesthetic values (simplicity, symmetry, coherence of limits), and experience itself incorporates, beyond the laboratory, trained qualitative perception (clinical, forensic, fine spectroscopy, "expert eye") (Ginzburg, 1992; Peirce, 1998; Ivanova & French, 2020). In a pragmatic–Bayesian key, one prioritises H if it maximises $\Pr(E \mid H) \cdot \mathcal{A}(H)$ under complexity penalty and correspondence constraint (Royall, 1997; Akaike, 1974; Schwarz, 1978; Rissanen, 1978).

This theoretical framework makes it possible to reread the commonly accepted historical itinerary of the origins of quantum theory according to a uniformly alternative interpretive criterion: Planck introduces $h$to fit black-body radiation (Planck, 1901); Einstein postulates light quanta for the photoelectric effect (energy–frequency linearity and threshold) (Einstein, 1905); Bohr imposes quantisation conditions and formulates the correspondence principle (Bohr, 1913); de Broglie proposes $\lambda = h/p$for matter (de Broglie, 1924); Heisenberg builds an algebraic kinematics of non-commuting observables (Heisenberg, 1927); Schrödinger establishes the wave equation (Schrödinger, 1926); Pauli introduces the exclusion principle and, later, the neutrino (Pauli, 1925, 1930). In all these cases, a finite collection of clues and a criterion of form guide the expansion of the hypothesis space (Ivanova & French, 2020; McAllister, 1996).

The aesthetic dimension in the discovery of the real is not taken here as a private preference of theoretical or methodological order: it is operationalised in a functional that rewards parsimony (few free parameters and auxiliary hypotheses), symmetry (invariances and conservation laws), coherence of limits (correct matching for $\hbar \to 0$, $v \ll c$, $k_B T \gg \hbar\omega$) and unification (integration of previously disjoint domains) (Noether, 1918/1971; Bohr, 1913; Ivanova & French, 2020; McAllister, 1996). This formalisation allows one to compare hypotheses that are equally coherent with the data and decide between them on the basis of explicit, replicable criteria (Royall, 1997; Akaike, 1974; Schwarz, 1978; Rissanen, 1978).

The discursive structure of the article is as follows: (i) section 2 presents the theoretical–methodological framework: definition of the indiciary paradigm, formalisation of abduction and specification of the aesthetic functional and its measures (Ginzburg, 1992; Peirce, 1998; Aliseda, 2006; Ivanova & French, 2020; McAllister, 1996); (ii) section 3 applies the protocol clues → conjecture → formalism → evaluation to canonical cases (Planck, 1901; Einstein, 1905; Bohr, 1913; de Broglie, 1924; Heisenberg, 1927; Schrödinger, 1926; Pauli, 1925, 1930); and (iii) finally, the conclusion gathers philosophical and pedagogical implications, theoretical–methodological limitations and a prospective research agenda. The ultimate aim is to offer an alternative narrative for the history of science that explains the quantum genesis without rhetoric of rupture, by means of an explicit economy of inferences guided by clues, evidence and criteria of form (Jaynes, 2003; MacKay, 2003; Lipton, 2004).

2. Theoretical–Methodological Framework of the Indiciary Paradigm

2.1. Minimal Ontology of the Discovery Problem

We shall call a *discovery problem* the quadruple (S, K, H, Θ), where:

- S is the set of clues: observations, local regularities, discontinuities or conceptual tensions extracted by salience (e.g. spectral tails, thresholds, discretisations).
- K denotes the available theoretical–metrological background: modelling schemes, instrumental standards, limits of validity and null models H_0 against which one contrasts (Peirce, 1998; Ginzburg, 1992).

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- H is a candidate hypothesis with explicit domain of application and boundary conditions (Aliseda, 2006).
- $\Theta$is the vector of parameters (physical and/or auxiliary) that govern the empirical–mathematical behaviour of H.

It is important to emphasise the limited scope of this minimal ontology. The quadruple (S, K, H, Θ)does not purport to capture the entirety of the investigator's creative experience, but only the decisional stratum in which, given a particular theoretical–metrological background K, hypotheses H are compared in the light of a set of clues S. The phenomenology of invention—images, intuitions, thought-habits, social contexts—lies largely outside this scheme; what is formalised here is only the epistemic interface where creativity crystallises into choices between models.

2.1.1. Comparability Assumptions

To avoid ambiguities:

1. Specification of the null. One fixes $H_0 \in K$(e.g. the relevant classical model) as an informational reference for all contrasts.
2. Domain and regime. Each Hdeclares its range of validity (e.g. v, T, v/c, dimensionless α) and the canonical correspondence limits (Bohr, 1913).
3. Identifiability. The parameters $\Theta$are identifiable at the available experimental resolution; constraints or priors are specified where appropriate.
4. Metrological stability. Instrumental error and conditions for repeatability are documented at the scale of the phenomenon.
5. Commutativity of limits. One evaluates $\lim_{\hbar\to 0}$, $\lim_{v/c\to 0}$, $\lim_{k_B T/\hbar\omega\to\infty}$, and other relevant limits, clearly indicating possible non-commutativities.

2.1.2. Notation and Domains

We denote by $\mathcal{H}$ the set of admissible candidate hypotheses. The operative evaluation functions are:

- Indicial weight $W(S; H, K)$.
- Aesthetic filter $\mathcal{A}(H)$.
- Fecundity $\Phi(H; K)$.
- Correspondence constraint $\mathcal{L}(H) \leq \varepsilon$ (maximum admissible asymptotic error).

With these, we write the decision rule:

$$H^* \in \arg\max_{H\in\mathcal{H}} (W(S; H, K))\, \mathcal{A}(H)\, \Phi(H; K) \text{subject to} \mathcal{L}(H) \leq \varepsilon.$$

2.2. Indicial Weight under Weak Signals

When the evidence is fragmentary and of low density, we quantify the advantage of Hover the null $H_0$by means of a penalised likelihood ratio (Royall, 1997):

$$W(S; H, K) \equiv \log \frac{L(S \mid H, K)}{L(S \mid H_0, K)} - \lambda\, \mathrm{pen}(H),$$

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where $L$is a likelihood (or a proxy such as an inverse residual-fit measure) and pen$(H) \in$ {AIC, BIC, MDL}controls effective complexity (Akaike, 1974; Schwarz, 1978; Rissanen, 1978). The hyperparameter $\lambda > 0$calibrates aversion to complexity.

Operational properties:

- Reparametrisation invariance. If $\Theta' = \psi(\Theta)$is bijective and smooth, W is invariant up to $O_p(1)$terms, preventing spurious "gains" through coordinate changes.
- Comparison of non-nested models. The penalised GLR allows comparison of H not nested within H_0; pen(H)prevents preference for redundant parametric freedom (Royall, 1997).
- Approximate additivity. For almost independent subsets $S = S_1 \cup S_2$:

$$W(S) \approx W(S_1) + W(S_2) - \lambda\,\Delta\text{pen},$$

which formalises the accumulation of heterogeneous clues.

- Naturalisation. A fact $E \in S$is naturalised by H when $\log[L(E \mid H, K)/L(E \mid H_0, K)] \gg 0$. Abduction justifies proposing Hif a small set of such Edecisively raises W(Peirce, 1998; Aliseda, 2006).
- Robustness with respect to K. Sensitivity analysis is required, varying H_0within K(reasonable alternative nulls) to verify the stability of the sign and magnitude of W.

Methodological note: when L is not available in closed form, one may use fit functions (e.g. weighted least squares in spectral domain) provided they preserve the ordering of preference among hypotheses.

2.3. Operationalised Epistemic Aesthetics

Epistemic aesthetics acts as a normative regulariser when several models tie in Wor when the evidence is equicoherent. We formalise it as:

$$\mathcal{A}(H) = e^{-\alpha\,\mathcal{C}(H)}\,(1 + \beta\,\mathrm{Sym}(H))\,e^{-\gamma\,\mathcal{L}(H)}\,(1 + \delta\,\mathcal{U}(H)),$$

with $\alpha, \beta, \gamma, \delta > 0$and four measurable components:

1. $\mathcal{C}(H)$: effective complexity

   — number of effective free parameters (k_{eff}), descriptive length (MDL) and burden of auxiliary hypotheses. Penalises overfitting and explanatory fragility (Akaike, 1974; Rissanen, 1978).

2. $\mathrm{Sym}(H)$: symmetry

   — dimension/rank of the group of invariances preserved or promoted by H, and number of derivable constants of motion (Noether, 1918/1971). Rewards structure and unification.

3. $\mathcal{L}(H)$: coherence of limits

   — relative asymptotic error with respect to classical regimes: $\hbar \to 0$, $v \ll c$, $k_B T \gg \hbar\omega$, etc. Guarantees continuity with K(Bohr, 1913).

4. $\mathcal{U}(H)$: unification

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— number (and weight) of phenomenological domains integrated by H under a common formalism with error below threshold; rewards inter-domain transfer (Ivanova & French, 2020; McAllister, 1996).

Selection of hyperparameters: $\alpha, \beta, \gamma, \delta$ encode the epistemic norms of a community and an epoch: tolerance for complexity, appreciation of symmetries, demand for correspondence and unificatory ambition. Their calibration is reported via sensitivity analysis and robustness checks.

This formulation does not equate aesthetic creativity with the functional $\mathcal{A}(H)$. The purpose is more modest and, at the same time, more sharply delimited: to make explicit those features of form that already operate, tacitly, in scientific practice as criteria of preference between hypotheses when the data do not suffice to fix a single option. The term $\mathcal{A}(H)$ should thus be understood as a *regulative idealisation*[1]: it provides a comparative metric for parsimony, symmetry, coherence of limits and unification, without thereby exhausting the phenomenological richness of the aesthetic experience of discovery.

Table 1. Components of $\mathcal{A}(H)$: measures and illustrative thresholds

Component	Operative meaning	Illustrative measures / reference thresholds
$\mathcal{C}(H)$	Effective complexity. Number of free parameters and auxiliary hypotheses, minimum descriptive length of the model (MDL).	$\mathcal{C}(H) \approx \log(\text{no. of parameters}) +$ code-length of H; penalty threshold $\lambda \in$
$\mathrm{Sym}(H)$	Symmetry. Dimension of the invariance group preserved or promoted by H; structural stability as per Noether's theorem.	$\dim G(H) +$ no. of constants of motion; threshold $\beta \in$
$\mathcal{L}(H)$	Coherence of limits. Consistency of H in canonical asymptotic regimes.	Relative asymptotic error in ($\hbar \to 0, v \ll c, k_B T \gg \hbar\omega$); threshold $\varepsilon \leq 10^{-2}$.
$\mathcal{U}(H)$	Unification. Capacity of H to integrate previously disjoint phenomenological domains under a single formalism.	Number of domains integrated with error < 5%; factor $\delta \in$

- The parameters $(\alpha, \beta, \gamma, \delta)$ are normative hyperparameters that weight, respectively, tolerance for complexity, valuation of symmetry, demand for correspondence and unificatory ambition.
- Thresholds λ and εcan be calibrated empirically or via hierarchical Bayesian adjustment by domain.
- Magnitudes are expressed in dimensionless units normalised with respect to K(the theoretical–metrological background).

2.4. Fecundity and the Programme Horizon

The fecundity of a hypothesis $H$is not reducible to its local fit: what matters is its prospective capacity within the background K. We model it as

$$\Phi(H; K) \simeq 1 + \eta(\Pi_{\text{pred}}(H; K) + \Pi_{\text{unif}}(H; K) + \Pi_{\text{tech}}(H; K)),$$

where $\eta > 0$scales prospective relevance and:

[1] We stress this point: what is proposed here is not a psychology of scientific creation, but a logical–methodological reconstruction of the epistemic function of certain criteria of form in contexts of empirical underdetermination.

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- $\Pi_{\text{pred}}(H;K)$: novel, testable prediction—for example, the number of statements E_{nov}such that $E_{\text{nov}} \notin K$, with likelihood ratio $L(E_{\text{nov}} \mid H,K)/L(E_{\text{nov}} \mid H_0,K) \geq \tau$ (Royall, 1997; Jaynes, 2003; MacKay, 2003).

- $\Pi_{\text{unif}}(H;K)$: unification—weighted count of phenomenological domains integrated by the same scheme (e.g. spectra + photoeffect), under domain-specific mean absolute error $\text{MAE}_d < \tau_d$ (Ivanova & French, 2020; McAllister, 1996).

- $\Pi_{\text{tech}}(H;K)$: technification potential—operative gains in calculation, instrumentation or simulation (e.g. new operators or measurement procedures).

Convention: the $+1$ shift prevents Φ from collapsing to zero in embryonic states, preserving comparability between incipient and mature hypotheses.

Computation guidelines: (1) normalisation by experimental effort for Π_{pred}; (2) weighting by domain relevance for Π_{unif}; (3) qualitative–quantitative recording of innovations in Π_{tech}.

2.5. Decision Rule and Formal Properties

We reaffirm the global decision rule:

$$H^* \in \arg\max_{H\in\mathcal{H}}(W(S;H,K))\,\mathcal{A}(H)\,\Phi(H;K)\text{subject to}\mathcal{L}(H) \leq \varepsilon.$$

2.5.1. Properties

(P1) Defeasibility (non-monotonicity)

New evidence S'can overturn the preference if it alters Wor forces readjustments in $\mathcal{L}$(Aliseda, 2006).

(P2) Dominance of clues

For $\Delta W \gg \log(\mathcal{A}_2\Phi_2/(\mathcal{A}_1\Phi_1))$, differences in Wdominate over $\mathcal{A}$and Φ(Royall, 1997).

(P3) Tie-breaking

If $\exp(W_1) \approx \exp(W_2)$(e.g. ratio in $[1-\delta_W, 1+\delta_W]$), decisions are made first by $\mathcal{A}$ and, if the tie persists, by Φ.

(P4) Admissibility qualification

The constraint $\mathcal{L}(H) \leq \varepsilon$ discards candidates with poor correspondence in limits (Bohr, 1913).

(P5) Reparametric invariance

$H$and its bijective reparametrisation $\psi(H)$ are decision-equivalent up to $o_p(1)$if W and $\mathcal{C}$ are correctly defined (Akaike, 1974; Rissanen, 1978).

(P6) Stability with respect to K

Sensitivity with respect to K(variants of H_0, metrological standards) must be analysed and stability ranges reported.

2.5.2. Operative Use

- Input: $S, K, \mathcal{H}, \varepsilon, \lambda, \alpha, \beta, \gamma, \delta, \eta$.

- Output: $H^*$and diagnostic: $W, \mathcal{A}, \Phi, \mathcal{L}$ disaggregated.

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- Stopping rule: stability of H^*under perturbations $\pm 10\%$in hyperparameters and under reasonable substitutions of H_0.

2.6. Cognitive Algorithm of *Musement*

Call *musement* the state of attentive exploration in which analogies and patterns are tested before formal consolidation (Peirce, 1998). The following replicable pipeline is proposed:

1. Salience (extraction of traces): identify in $S$the systematic residuals under H_0(tails, thresholds, discretisations, unexpected correlations).

2. Analogisation (pre-model): map traces onto morpho-semantic schemes: discretisation, duality, saturation, broken/preserved symmetry.

3. Conjecture (generation of H): formulate H with minimal Θ such that it naturalises $E \in S$: maximise $\log L(E \mid H, K) - \log L(E \mid H_0, K)$.

4. Limits check ($\mathcal{L}$): evaluate asymptotics $\hbar \to 0$, $v/c \to 0$, $k_B T/\hbar\omega \to \infty$and other case-specific limits; set ε.

5. Symmetries (Noether): identify invariances and constants of motion; quantify $\mathrm{Sym}(H)$(Noether, 1918/1971).

6. Projection (fecundity Φ): list novel predictions, inter-domain transfers and technification potential; estimate $\Pi_{\mathrm{pred}}, \Pi_{\mathrm{unif}}, \Pi_{\mathrm{tech}}$.

7. Selection (decision): compute $W, \mathcal{A}, \Phi$; apply the rule with $\mathcal{L} \leq \varepsilon$; document sensitivity and robustness.

That is:

- Case input: (S, K, H_0).
- Expected output: $\{H^*, W, \mathcal{A}(\mathcal{C}, \mathrm{Sym}, \mathcal{L}, \mathcal{U}), \Phi\}$.
- Checks: limits, ties, sensitivity in $\lambda, \alpha, \beta, \gamma, \delta, \eta$.

In Peircean terms, these steps describe the dynamics by which an initially diffuse semiosis is oriented towards a genuine logical interpretant only when the appropriate *kairos* is reached: neither earlier—when emotional or energetic interpretants suffice for the "rough use" of the sign—nor later—when the interpretant is already fixed and merely reactivated by habit (Ramírez Suárez, 2007, pp. 289–293). The musement algorithm can thus be read as a minimal formalisation of this search for triadic synchrony: the indiciary-abductive logic does not seek to mechanise invention, but to make explicit the conditions under which a particular experimental trace becomes a sign of opportune occasion for the introduction of a new hypothesis.

2.7. Operational Differences Between Indiciary and Galilean–Newtonian Paradigms

We fix the following operational contrasts:

- Unit of analysis
  - Indiciary–abductive: informative singular case (local traces).
  - Galilean–Newtonian: populations and repeatable regularities.
- Dominant inference
  - Indiciary–abductive: abduction with non-monotonic decision; $W + \mathcal{A} + \Phi$.

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- Galilean–Newtonian: induction (law) → deduction (prediction).

- Evidence
  - Indiciary–abductive: weak signals and discontinuities; penalised GLR (Royall, 1997; Akaike, 1974; Schwarz, 1978; Rissanen, 1978).
  - Galilean–Newtonian: robust series, repeated measurements, well-behaved Gaussian errors.

- Epistemic aesthetics
  - Indiciary–abductive: explicit normative filter $\mathcal{A}$(Ivanova & French, 2020; McAllister, 1996).
  - Galilean–Newtonian: tacit or ex post use of criteria of form.

- Correspondence
  - Indiciary–abductive: hard constraint $\mathcal{L} \leq \varepsilon$(Bohr, 1913).
  - Galilean–Newtonian: implicit continuity across scales within the same regime.

- Typical outcome
  - Indiciary–abductive: theoretical prototypes that open a research programme.
  - Galilean–Newtonian: universal laws with extensive validation.

- Governance of symmetries
  - Indiciary–abductive: symmetries as heuristic and metric (Sym).
  - Galilean–Newtonian: symmetries as foundation of already established invariances (Noether, 1918/1971).

2.8. Objections

Objection 1: aesthetic reductionism

Reply: we distinguish between the phenomenology of invention and the normativity of decision. The model proposed does not aim to describe how one "has" a new idea, nor how the beauty of a theory is lived internally, but exclusively how certain features of form (parsimony, symmetry, coherence of limits, unification) can intervene in the justifiable selection between hypotheses that are empirically equicoherent. In this sense, $\mathcal{A}(H)$is not an ontology of aesthetic experience, but a tool for making auditable the criteria of form that scientific communities already employ—albeit implicitly—when preferring some models over others.

Objection 2: arbitrariness of hyperparameters $\alpha, \beta, \gamma, \delta$

Reply: far from being "metaphysical" constants, these weights encode values of epoch and community (tolerance for complexity, appreciation of symmetry, demand for correspondence, unificatory ambition) and are therefore open to historical analysis and empirical adjustment. Instead of naturalising a particular aesthetic taste, the formalisation allows us to study how these weights evolve over the history of physics and how they relate to concrete decisions of theory acceptance or rejection.

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Objection 3: dependence on background K and null H_0

Reply: cross-paradigm checks (substituting H_0 by reasonable nulls; reporting the range of H^*; incorporating predictive robustness tests) (Royall, 1997; Jaynes, 2003; MacKay, 2003).

Objection 4: aesthetic overfitting of the historical narrative

Reply: the use of $\mathcal{A}(H)$ in this work is subject to several safeguards: (i) it is applied only in clearly identified situations of incomplete evidence; (ii) it is always accompanied by analysis of indicial weight $W(S;H,K)$ and fecundity $\Phi(H;K)$; and (iii) it is cross-checked against standard historiographical sources, so that any aesthetic reconstruction remains anchored in documented decisions rather than retrospective projection.

Objection 5: circularity (choosing "what works" as beautiful)

Reply: pre-register criteria of form and their weights; demand empirical successes independent of the symmetry metric to confirm them.

Objection 6: instability under reparametrisation

Reply: use MDL and invariant criteria; demonstrate equivalence under $\psi: \Theta \to \Theta'$ (Akaike, 1974; Rissanen, 1978).

Conditions for responsible use: (i) $\mathcal{L}$ as a hard constraint; (ii) systematic reporting of W, $\mathcal{A}$, Φ disaggregated; (iii) ex ante predictions to audit Φ.

3. Application of the Indiciary–Abductive Protocol to Canonical Episodes in the Genesis of Quantum Theory (1900–1930)

The foregoing objections and safeguards delimit the scope and conditions of use of the proposed model. The indiciary–abductive framework thereby becomes a reconstructive tool for a philosophical history of science, rather than a mere psychology of invention or an aesthetic reduction of scientific practice: $W(S;H,K)$ fixes the indicial strength of the data, $\mathcal{A}(H)$ makes explicit the criteria of form already operative in the community (Ginzburg, 1992; Ivanova & French, 2020; McAllister, 1996), $\Phi(H;K)$ introduces the programme horizon, and $\mathcal{L}(H)$ secures continuity with classical regimes. Once the audibility criteria—component disaggregation, sensitivity analysis, robustness checks, reparametrisation invariance—are established, it becomes possible to move from the methodological to the historical–philosophical plane.

In this sense, to speak of a philosophical history of science is more than to recount successful episodes in the development of contemporary physics: it involves reconstructing, at a microhistorical scale, regularities in the logical procedures of discovery that recur under different empirical configurations and point back to a shared background of epistemic and aesthetic values (Ginzburg, 1992; Kuhn, 1978; Koyré, 1968). Applying the indiciary paradigm to these quantum cases not only illuminates how particular hypotheses are locally justified, but also how a "spirit of the age"—made of controlled neophilia, confidence in mathematization, sensitivity to form and invariance—spontaneously and operatively enters scientific practice, modulating which anomalies are perceived as relevant traces, which conjectures are considered admissible, and which criteria of form acquire normative force (Hacking, 2012; Daston & Galison, 2007). The proposed model of epistemic aesthetics thus allows us to move from a collection of isolated episodes to the identification of a pattern of rationality characteristic of the quantum germ: an indiciary–abductive logic in which the emerging culture of twentieth-century physics is implicitly yet effectively encoded in the rules of theory choice.

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The procedure adopted is, in this sense, deliberately microhistorical: following Ginzburg's inspiration, quantum episodes are not treated as large retrospective syntheses, but as local scenes in which small phenomenological mismatches—a spectral curvature, a frequency threshold, a regularity in emission lines—function as traces capable of reordering an entire theoretical field (Ginzburg, 1992). Far from being mere isolated "cases", these indiciary cores enable us to detect regularities in the modes of reasoning and seeing characteristic of a given scientific culture: what we call here the indiciary–abductive paradigm is precisely the logical–aesthetic crystallisation of a certain *zeitgeist*, in which fine sensitivity to the shape of data and the readiness to accept minimal discretisations become spontaneous operators of discovery.

The aim of the next section is precisely to test the explanatory capacity of the indiciary–abductive paradigm on concrete episodes of quantum genesis. The point is not to impose an aesthetic reading upon the facts, but to show how, in each case (Planck, Einstein, Bohr, de Broglie, Heisenberg, Schrödinger, Pauli), a discrete set of clues, a given theoretical background and a criterion of form influenced the selection of the hypotheses eventually adopted. The sequence follows the usual chronological order, but the reading is reconfigured: each episode is analysed as a transition $S \to H$ subject to $W, \mathcal{A}, \Phi$ and $\mathcal{L}$. Under these coordinates, the Planck case functions as the inaugural laboratory of the method.

3.1. Planck (1900–1901): the Black-Body Anomaly as Indiciary Matrix

The starting point of the quantum itinerary is not a new theory, but a set of clues locally resistant to classical physics. The spectrum of black-body radiation, reconstructed experimentally by Lummer, Pringsheim and Rubens between 1898 and 1900, displayed a morphology incompatible with the predictions of Rayleigh–Jeans in the ultraviolet regime and Wien's law in the infrared. The systematic failure of both approximations constitutes, in Ginzburg's sense, a trace: a "minor detail" that does not fit within the prevailing explanatory network (Ginzburg, 1992).

Formally, the set of clues

$$S_{\text{Planck}} = \{\sigma_{\text{UV}}, \sigma_{\text{IR}}, \nu_{\text{peak}}, \kappa_{\text{tail}}\}$$

comprises: (i) ultraviolet divergence; (ii) infrared insufficiency; (iii) systematic shift of the maximum; and (iv) specific features of the spectral curvature. The background K—classical thermodynamics, wave electrodynamics and Boltzmannian statistics—fixes expectations of continuity and energy distributions with unlimited degrees of freedom.

The indicial salience of S_{Planck} is high because no reasonable combination of classical auxiliary hypotheses can simultaneously fit UV + IR + peak + curvature. Planck himself acknowledges this phenomenological resistance in his 1900 memoir, where he describes his intervention as an "act of desperation" (Planck, 1901), a gesture that, in Peircean logic, could be interpreted as the capture of semiotic *kairos*: the moment when an experimental trace can no longer be relegated to noise and demands a new interpretant.

Under the pipeline of the indiciary–abductive logic, the transition $S \to H$ takes the following form:

1. Conceptual analogisation (*musement*): the discordant statistical behaviour invites an attempt at restricting energy exchange between oscillators—a formal move which, although initially ad hoc, preserves continuity with the Hamiltonian framework (Peirce, 1998; Aliseda, 2006).

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2. Conjecture H. H_{Planck}: $E = nh\nu$: this is a minimal conjecture: it does not postulate a corpuscular nature of light nor universal field quantisation; it limits discretisation to exchange between resonators and radiation.

3. Indicial weight $W(S; H, K)$: the combined UV+IR fit yields a massive increase in the log-likelihood ratio, so that

$$W(S_{\text{Planck}}; H_{\text{Planck}}, K) \gg 0,$$

especially when one penalises the complexity of alternative hypotheses (Akaike, 1974; Schwarz, 1978; Royall, 1997; Rissanen, 1978).

4. Aesthetic value $\mathcal{A}(H)$: in Planck's case, parsimony is high (a single fundamental parameter h); structural symmetry is neutral (no additional invariances are invoked); coherence of limits $\mathcal{L}(H)$ is acceptable: as $h \to 0$, the formalism recovers the Rayleigh–Jeans distribution in the classical regime. These conditions yield a reasonably favourable $\mathcal{A}(H)$ (Ivanova & French, 2020; McAllister, 1996).

5. Fecundity $\Phi(H; K)$: in this episode, fecundity is modest but real: although the hypothesis does not predict new phenomena beyond the black-body spectrum, it canonises the parameter *h*and opens a programme horizon of quantisation for later episodes.

6. Correspondence constraint $\mathcal{L}(H)$: Planck does not formulate an explicit principle, but his construction satisfies the embryonic version of what Bohr would systematise in 1913: in classical limits, the theory returns to itself and respects the expected continuous behaviour.

The overall balance can be summarised comparatively:

Component	Relative value	Comment
W	Very high	Integrated UV + IR fit with no divergence.
$\mathcal{A}$	Medium–high	Strong parsimony; correct limits.
Φ	Low—medium	Little novel prediction, but high structural importance.
$\mathcal{L}$	Satisfied	Classical limit respected ($h \to 0$).

established in §2 under soft constraints. The Galilean–Newtonian paradigm—centred on continuity, mechanical analogy and quantitative deduction—did not possess a mechanism capable of absorbing all four clues without compromising the classical statistical structure. Planck's gesture, though perhaps not fully conscious of its scope, exemplifies the aesthetic–epistemic potential of the indiciary–abductive paradigm: theory is born not from a systematic programme, but from the fine reading of local anomalies and a stylistically modest conjecture that makes them decodable.

In this sense, Planck inaugurates the quantum sequence not as a doctrinal founder, but as the first indiciary operator at microhistorical scale: an intellectual potency, embedded in the meaningful flow of information and situated within a particular culture, who, in the theoretical physics of his time, perceives in a spectral anomaly not an instrumental failure nor a local exception, but a trace—a sign of occasion—that requires a minimal yet normatively fertile change in the space of contemporary physical hypotheses.

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3.2. Einstein (1905): the Photoelectric Effect as an Indiciary Case of Energy–Frequency Linearity

Einstein's 1905 contribution to the photoelectric problem constitutes the second major episode in which the indiciary–abductive paradigm shows its reconstructive capacity in the historical germ of quantum theory. Unlike the Planck case, where the anomaly lay in the global shape of the black-body spectrum, here the empirical clues are local and robust: (i) existence of a frequency threshold for electron emission; (ii) independence of the maximum kinetic energy from the intensity of the incident light; and (iii) practically instantaneous response of the emission. These features had been stably documented since Lenard's experiments, but they could not be accommodated within classical wave theory (Lenard, 1902; see also Pais, 2005 for historical analysis).

(a) Clues S_{1905}

We can schematise the relevant clue set as:

$$S_{1905} = \{\nu_{\text{thr}}, K_{\max}(\nu) \text{ linear}, \text{independence from intensity}, \Delta t \approx 0\}.$$

In the framework of §2, these clues exhibit high salience because they directly contradict the predictions of the classical background K_{clas}:

- According to Maxwell–Lorentz, the energy absorbed by an electron depends on intensity, not only on frequency;
- Emission should display delays attributable to gradual energy accumulation;
- No rigid threshold is predicted, since absorbed energy can be arbitrarily small but cumulative.

Thus S_{1905} constitutes a highly anomalous set under $H_0 = H_{\text{wave}}$.

(b) Indicial weight $W(S_{1905}; H, K)$

Using the general model:

$$W(S; H, K) \equiv \log \frac{L(S \mid H, K)}{L(S \mid H_0, K)} - \lambda \,\text{pen}(H),$$

we obtain, in the Einstein case, a structure in which the likelihood ratio strongly favours a hypothesis that incorporates discrete energy quanta. The key point is that, under H_{wave}:

- $\Pr(K_{\max}(\nu) \text{ approximately linear}) \approx 0$,
- $\Pr(\nu_{\text{thr}} \text{ strict}) \approx 0$,
- $\Pr(\Delta t \approx 0) \approx 0$.

This implies

$$W(S_{1905}; H_{\text{quant}}, K_{\text{clas}}) \gg 0.$$

(c) Einstein's hypothesis as abductive conjecture

Einstein postulates that radiation of frequency ν is composed of energy packets:

$$E = h\nu,$$

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a proposition that, beyond explaining the clues, immediately yields the experimentally verified equation:

$$K_{\max} = h\nu - \varphi,$$

where φ is the work function of the material.

From the standpoint of §2, this hypothesis is a classic Peircean abduction: it selects the simplest cause capable of unifying heterogeneous clues. Einstein does not derive $E = h\nu$from any pre-existing formalism, nor does he justify it by statistical optimisation: he conjectures it as the most plausible explanation for a set of signals that the dominant model could not absorb.

(d) Aesthetic evaluation $\mathcal{A}(H_{\text{quant}})$

As defined:

$$\mathcal{A}(H) = e^{-\alpha\,\mathcal{C}(H)}\,(1 + \beta\,\mathrm{Sym}(H))\,e^{-\gamma\,\mathcal{L}(H)}\,(1 + \delta\,\mathcal{U}(H)),$$

we obtain in the Einstein case:

1. Complexity $\mathcal{C}(H)$

   The hypothesis introduces a single new parameter (the quantum h) and a single postulate (linearity $E = h\nu$):

   $$\mathcal{C}(H_{\text{quant}}) \approx 1$$

under descriptive-length criteria.

2. Symmetry $\mathrm{Sym}(H)$

   Einstein does not introduce strong structural ruptures; he preserves linear functional form and respects classical invariances. Thus:

   $$\mathrm{Sym}(H_{\text{quant}}) > 0,$$

but moderate.

3. Coherence of limits $\mathcal{L}(H)$

   In the regime $\hbar \to 0$(or equivalent macroscopic high-intensity limits), the hypothesis reproduces classical wave behaviour:

   $$\mathcal{L}(H_{\text{quant}}) \approx 0$$

(high coherence of limits).

4. Unification $\mathcal{U}(H)$

   The postulate unifies two previously separate domains:

(i) energetic description of radiation, and
(ii) electronic emission.

Therefore:

$$\mathcal{U}(H_{\text{quant}}) > 0.$$

Taken together, these values yield a high $\mathcal{A}(H_{\text{quant}})$.

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(e) Fecundity $\Phi(H_{\text{quant}}; K)$

The general term

$$\Phi(H; K) \simeq 1 + \eta(\Pi_{\text{pred}} + \Pi_{\text{unif}} + \Pi_{\text{tech}})$$

takes particularly high values here:

- Prediction: Einstein's equation for the photoelectric effect is experimentally verified in numerous materials.
- Unification: it connects optics, surface thermodynamics and electronic phenomena.
- Technological projection: it anticipates photocells, detectors and primitive quantum devices.

Thus

$$\Phi(H_{\text{quant}}; K_{\text{clas}}) \gg 1.$$

(f) Model selection H^*

Integrating,

$$H^* \in \arg\max_{H \in \mathcal{H}} (W(S; H, K))\, \mathcal{A}(H)\, \Phi(H; K) \text{ subject to} \mathcal{L}(H) \leq \varepsilon,$$

we obtain, for the set of hypotheses available in 1905,

$$H^* = H_{\text{quant}}^{\text{Einstein}},$$

since it simultaneously maximises:

- indicial strength,
- descriptive economy,
- coherence of limits,
- unificatory capacity,
- and predictive fecundity.

In indiciary–abductive terms: from a set of local signals Einstein extracts a globally plausible structure—energy discretisation—with minimal complexity and maximal explanatory efficacy. From our perspective, the 1905 contribution can be read as the fixation of a logical interpretant at the precise point of semiosis: neither earlier (when the photoelectric data were still marginal), nor later (when quantum ontology would already have crystallised).

(g) Historiographical and epistemological reading

The historical–philosophical reconstruction shows that the value of Einstein's proposal does not derive from a metaphysical principle about the nature of light, but from its optimal fit within the hypothesis space defined by laboratory clues. In this sense, his intellectual move clearly meets the criteria of the indiciary paradigm: sensitivity to anomalies, abductive transition and operative aesthetic selection.

Historiographical literature converges in noting that the 1905 article does not introduce quantum theory as a new ontological framework, but as a local inferential resource intended to resolve a cluster of irregularities (Pais, 2005; Kuhn, 1978). The later ontological

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elevation of the quantum belongs to other phases of quantum development, but the heuristic analysed here is strictly indicial: one starts from empirical traces and forms a minimal yet effective conjecture.

The transition from Planck to Einstein displays the operational consolidation of the indiciary–abductive paradigm in the germ of quantum theory. If in 1900 the reading of spectral anomalies led Planck to introduce a minimal discretisation of energy exchange—a local modification designed to save the classical structure—, the Einstein episode shifts the focus from a global spectral irregularity to a finer set of independent experimental signals: rigid threshold, energy–frequency linearity and intensity independence. This shift is not a change of method, but an intensification of the same inferential pattern: from the analysis of a "global" trace, a sign (the black-body curve), one moves to the reading of "local traces" (point-like photoemission) under the same regime of interpretation. In terms of the epistemic aesthetics developed here, Einstein radicalises the gesture inaugurated by Planck: he takes as starting point a discrete packet of clues that resist wave-background expectations, triggers an abductive transition towards the simplest hypothesis capable of unifying them, and immediately subjects it to criteria of parsimony, coherence of limits and fecundity. Thus the photoelectric effect is not a rupture with respect to the Planck laboratory, but its natural continuation: the indiciary paradigm leaves its embryonic condition and acquires explicit form in the genesis of quantum mechanics, showing that the quantum "revolution" does not arise from grand deductive programmes, but from the ability to read minimal signals and articulate low-cost conjectures that reorganise the available theoretical space.

3.3. Bohr (1913): the Indiciary Leap towards Quantum Architecture

The historical–philosophical sequence from Planck to Einstein can be read, retrospectively, as the establishment of a double methodological threshold: on the one hand, the legitimisation of local anomaly as "trace" or "sign of occasion" with normative capacity to reorganise a theoretical domain; on the other, the acceptance that a minimal conjecture—if parsimonious, limit-coherent and unificatory—can prevail over a consolidated background. With both criteria already operative in early twentieth-century theoretical physics, the scene was set, by 1913, for a new indiciary leap: if Planck had introduced discretisation in radiative interaction, and Einstein had transferred that discretisation to electronic emission, Bohr now faced a set of deeper structural clues—spectral lines, combination rules, electronic stability—whose resistance to classical models was more structural than merely phenomenological. At this point, the indiciary–abductive paradigm assumes a new form: it is no longer simply a matter of fitting curves or explaining thresholds, but of reorganising theoretical architecture via a hypothesis that incorporates discretisation, orbital dynamics and correspondence with classical physics. Bohr thus acts as the third indiciary operator in the germ of quantum theory, recognising in the fine regularity of spectra not mere anomalies, but structural traces that demand a quantised conjecture endowed with programme horizon. The indiciary–abductive paradigm is here applied to a different kind of trace: not isolated "minor details", but systematic patterns whose normative relevance stems precisely from their persistence and from their capacity to resist several classical models at once (Bohr, 1913; Kuhn, 1978; Pais, 2005).

(a) Clues S_{1913}

We can schematically represent the relevant set S_{1913} as:

1. Atomic stability: in Rutherford's model, an accelerated electron in orbit should radiate continuously and collapse in very short times. The existence of stable atoms is thus a structural anomaly rather than a merely quantitative one.
2. Balmer–Rydberg rules: hydrogen frequencies satisfy relations of the form

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$$\nu = R(\frac{1}{n_1^2} - \frac{1}{n_2^2}),$$

with $n_1, n_2 \in \mathbb{N}$and $R$the Rydberg constant, suggesting discrete energy levels (Kuhn, 1978).

3. Combination regularities: lines are organised into series and combinations that point to level differences rather than absolute frequencies.
4. Intensities and selection: certain transitions are allowed and others forbidden, suggesting implicit selection rules.
5. The background K—Rutherford's model, classical electrodynamics and early "old quantum" attempts—was committed to a continuous dynamics of charges and fields. Under this K, the set S_{1913}cannot be absorbed without severe tensions: neither stability, nor Balmer–Rydberg-type discrete formulae, nor combination rules admit a direct derivation.

(b) Indicial weight $W(S_{1913}; H, K)$

In the general framework,

$$W(S; H, K) \equiv \log \frac{L(S \mid H, K)}{L(S \mid H_0, K)} - \lambda \, \mathrm{pen}(H),$$

where H_0 represents the best available classical candidate (Rutherford + standard electrodynamics). For S_{1913}, the likelihood under H_0 is low:

- Atomic stability is not explained without ad hoc hypotheses regarding radiation;
- Balmer–Rydberg formulae appear as purely empirical correlations;
- Combination and selection regularities lack explicit theoretical grounding.

By contrast, a hypothesis that introduces discrete stationary levels and quantised transitions can simultaneously accommodate stability and observed frequencies. In abbreviated notation:

$$W(S_{1913}; H_{\mathrm{Bohr}}, K) \gg 0,$$

when H_{Bohr} imposes stationary orbits and a quantisation condition, whereas H_0fails to explain stability + spectra jointly.

(c) Hypothesis H_{Bohr}as structural conjecture

Bohr's proposal can be condensed into two main blocks (Bohr, 1913):

1. Postulate of stationary states: only certain allowed orbits are dynamically stable; in them the electron does not radiate.
2. Quantisation condition: the angular momentum of the electron in each orbit satisfies

$$mvr = n\hbar,$$

with $n \in \mathbb{N}$; spectral frequencies are then obtained as energy differences between stationary states:

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$$h\nu = E_{n_2} - E_{n_1}.$$

From the standpoint of indiciary–abductive logic, H_{Bohr}appears as a minimal structural conjecture designed to naturalise S_{1913}: it preserves the Coulombic core of the dynamics, introduces discretisation only in the set of admissible orbits and translates empirical Balmer–Rydberg formulae into level differences. It is not derived from a more general formalism, but proposed as the most plausible explanation for a pattern of spectral and stability traces that the classical model could not absorb in a theoretically structured way.

(d) Aesthetic evaluation $\mathcal{A}(H_{\text{Bohr}})$

According to the functional

$$\mathcal{A}(H) = e^{-\alpha\,\mathcal{C}(H)}\,(1 + \beta\,\text{Sym}(H))\,e^{-\gamma\,\mathcal{L}(H)}\,(1 + \delta\,\mathcal{U}(H)),$$

we can estimate the components for H_{Bohr}:

1. Complexity $\mathcal{C}(H_{\text{Bohr}})$**:** several new elements are introduced (stationary states, angular-momentum quantisation, transition rules), so that

$$\mathcal{C}(H_{\text{Bohr}}) > \mathcal{C}(H_{\text{quant}}^{\text{Einstein}}) > \mathcal{C}(H_{\text{Planck}}).$$

Nevertheless, descriptive length remains moderate: a bounded set of postulates yields a large family of spectral lines (Akaike, 1974; Rissanen, 1978).

2. Symmetry $\text{Sym}(H_{\text{Bohr}})$**:** Bohr exploits the rotational symmetry of the Coulomb problem and preserves the structure of circular orbits analogous to Keplerian ones. The quantisation condition $mvr = n\hbar$respects that underlying symmetry, so that

$$\text{Sym}(H_{\text{Bohr}}) > 0,$$

with significant weight (Noether, 1918/1971; Ivanova & French, 2020).

3. Coherence of limits $\mathcal{L}(H_{\text{Bohr}})$**:** in the limit $n \to \infty$, level differences become small and behaviour approaches the classical continuum; moreover, in macroscopic regimes radiation recovers standard wave descriptions. Hence

$$\mathcal{L}(H_{\text{Bohr}}) \approx 0$$

in domains where classical physics is expected to hold.

(e) Unification and fecundity $\Phi(H_{\text{Bohr}}; K_{\text{clas}})$

Within the general scheme

$$\Phi(H;K) \simeq 1 + \eta(\Pi_{\text{pred}}(H;K) + \Pi_{\text{unif}}(H;K) + \Pi_{\text{tech}}(H;K)),$$

Bohr's model scores significantly higher than both Planck's and Einstein's earlier proposals.

On the predictive side, once the frequency condition and the selection rule $\Delta n = \pm 1$are fixed, the model reproduces with high accuracy the Balmer series and, more generally, the Rydberg–Ritz regularities for hydrogen-like spectra. In our notation, $\Pi_{\text{pred}}(H_{\text{Bohr}}; K_{\text{clas}}) \gg$

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0, since the theory does not merely interpolate existing data but yields a compact formula for the entire spectral series.

On the unificatory side, H_{Bohr}ties together three previously disjoint domains:

(i) classical Keplerian dynamics of central forces;

(ii) empirical spectroscopy (Balmer, Rydberg, Ritz); and

(iii) the emerging quantum of action as a structural constant rather than a mere fitting parameter.

This translates into a substantial positive contribution $\Pi_{\text{unif}}(H_{\text{Bohr}}; K_{\text{clas}}) > 0$.

From the technological/programmatic point of view, Bohr's model inaugurates a research programme on atomic structure: it provides a platform for discussing multi-electron atoms, chemical valence and periodicity. Even if later superseded, this horizon justifies taking $\Pi_{\text{tech}}(H_{\text{Bohr}}; K_{\text{clas}})$as moderately positive.

Collecting these contributions, we obtain

$$\Phi(H_{\text{Bohr}}; K_{\text{clas}}) \gg 1,$$

which, combined with a high indicial weight $W$and a reasonably favourable aesthetic value $\mathcal{A}$, explains why the community eventually converged on Bohr's scheme as the default interpretive framework for atomic phenomena in the 1910s.

In summary, for Bohr we can schematically represent the evaluation vector as

$$(W, \mathcal{A}, \Phi, \mathcal{L})\ H_{\text{Bohr}} \approx \text{(very high, medium–high, very high, satisfied)}.$$

The model remains semiclassical and dynamically unstable in a strict sense, but from the standpoint of the indiciary-abductive logic it clearly dominates the available competitors.

(f) Indiciary–abductive reading of Bohr's move

Read through the lens of the paradigm introduced in sections 1–2, Bohr's contribution is not simply the imposition of quantisation rules on classical orbits. It exemplifies a mature phase of the indiciary–abductive logic:

1. A structured set of spectral indices S_{Bohr} is recognised not as an annoying detail, but as a system of structural traces encoding information about the internal architecture of the atom.
2. The hypothesis H_{Bohr} is not derived deductively from K_{clas}. It is abduced as the simplest and most coherent way to make the spectral regularities "natural", by postulating stationary states and quantised transitions constrained by a correspondence requirement.
3. The aesthetic filter $\mathcal{A}(H_{\text{Bohr}})$ operates by rewarding:

   – a minimal extension of the classical formalism (only one new quantum condition, plus the frequency–transition link);

   – a robust correspondence behaviour in the high-n limit;

   – a high degree of compression of disparate regularities into a single scheme.

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4. The programme-opening character of the model is captured by $\Phi(H_{\text{Bohr}}; K_{\text{clas}})$: the hypothesis is not a local trick but the starting point of a lineage of further developments (Sommerfeld, old quantum theory, and, indirectly, wave and matrix mechanics).

In this sense, Bohr appears as the third major indiciary operator in the genesis of quantum theory. Where Planck had responded to a global spectral anomaly and Einstein to local threshold phenomena, Bohr responds to structural regularities in line spectra, reading them as traces of a hidden architecture and legitimising a non-classical organisation of orbital dynamics.

3.4. de Broglie (1923–1924): matter waves as indicial bridge between orbits and radiation

By the time Bohr's model had consolidated its status as the default interpretive framework for atomic structure, the indiciary–abductive pattern that had guided Planck and Einstein had acquired a new level of structural ambition. Global anomalies in thermal radiation (Planck), robust local thresholds in the photoelectric effect (Einstein), and finely grained spectral regularities (Bohr) had all been read as traces demanding minimal but normatively fertile departures from the classical background. What remained unresolved, however, was the deep conceptual asymmetry between the corpuscular language used to describe electrons in atoms and the wave language that continued to govern optics. It is at this juncture that de Broglie's intervention must be located: not as an isolated stroke of genius, but as the next step in a historical trajectory where the same indicial sensibility—to discrete patterns, stability conditions, and formal symmetries—drives an abductive conjecture that fuses Bohr's quantised orbits with the wave phenomenology of light. In this sense, de Broglie radicalises the emerging zeitgeist of early quantum theory: instead of merely quantising classical structures, he reads their discrete signatures as evidence for an underlying wave-like guidance common to matter and radiation.

The "fourth movement" in our indicial–abductive reconstruction is de Broglie's proposal of matter waves. Historically, it marks a shift from accommodating quantisation within a largely classical kinematics to reinterpreting quantisation itself as the surface manifestation of a deeper wave structure. Philosophically, it extends the indicial paradigm from local anomalies and structural regularities to a more ambitious symmetry argument: if Planck, Einstein and Bohr have progressively elevated the quantum of action from a fitting parameter to a structural constant of atomic dynamics, then the coexistence of discrete orbital conditions and robust wave phenomena invites an abductive synthesis at the level of representation.

(a) Indicia S_{dB}: between stable integers and wave interference

By the early 1920s, a characteristic constellation of clues was in place:

1. Bohr's quantisation had established a tight link between orbital stability and integral multiples of h: only discrete angular momenta $L = n\hbar$ensured stationary orbits with the observed energy levels.

2. Spectroscopic regularities (Rydberg–Ritz, combination rules) continued to display a striking arithmetic structure suggestive of underlying mode conditions rather than arbitrary parameters.

3. Wave optics maintained its full descriptive and predictive power for interference and diffraction: extended, coherent wave fields produced fringe patterns in strict accordance with classical wave equations.

4. The energy–frequency relation $E = h\nu$, already exploited by Einstein for light quanta and by Bohr for transition frequencies, had become entrenched as a cross-domain law for radiation.

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Taken together, these elements form a set of indicia that we can summarise as

$$S_{\mathrm{dB}} = \{\text{quantised orbits, stable integers } n,\ E = h\nu,\ \text{wave interference patterns}$$

Under the classical background K_{clas}, these traces remain conceptually disjoint. Matter is still described as localised point-like entities following trajectories; waves are extended, delocalised fields obeying superposition and interference. The very success of Bohr's model accentuates this tension: the integers $n$appear "by decree", with no dynamical rationale beyond the quantisation postulates.

Within the indicial–abductive framework, this tension is itself an index: a sign that the current representational split between particles and waves may be masking a deeper continuity.

(b) Abductive move and hypothesis H_{dB}: assigning a wavelength to matter

De Broglie's central conjecture can be seen as an attempt to render the Bohr integers natural by reinterpreting them as wave-number conditions. The key hypothesis is that to every material particle with momentum $p$there corresponds a wavelength λgiven by

$$\lambda = \frac{h}{p}.$$

This proposal does not arise as a deductive consequence of any established formalism; it is an abductive move that seeks to unify two previously disjoint families of indicia:

- the discrete orbital conditions and stability patterns captured by Bohr's model, and
- the continuous wave phenomena of interference and diffraction characteristic of radiation.

If light quanta, originally conceived as "particles", exhibit interference under suitable conditions, and if electrons and other material quanta exhibit discretisation patterns reminiscent of standing waves, then postulating a common wave-like substratum provides a simple, symmetry-driven explanation of both. In our notation, $H_{\mathrm{dB}}$is conspicuously low in complexity: it introduces a single relation linking a new wavelength $\lambda$to the already familiar dynamical quantity p, without inflating the ontology.

From the standpoint of the indicial logic, de Broglie's abductive step can be summarised as:

$$S_{\mathrm{dB}} \rightsquigarrow H_{\mathrm{dB}}\text{: "interpret Bohr's integers as mode numbers of a matter wave".}$$

(c) Evaluation via W, $\mathcal{A}$, $\Phi$and $\mathcal{L}$

At the moment of its proposal, the indicial weight $W(S_{\mathrm{dB}}; H_{\mathrm{dB}}, K_{\mathrm{clas}})$is largely theoretical rather than experimental. The hypothesis is constructed so that, if one interprets Bohr's quantisation condition as a requirement of single-valuedness for a wave along a closed orbit, the observed energy levels emerge as the eigenvalues of a standing-wave problem. The decisive empirical boost to Wcomes later, with Davisson–Germer's and G. P. Thomson's experiments on electron diffraction, which render classical corpuscular expectations for scattering patterns extremely unlikely. Retrospectively, this drives

$$W(S_{\mathrm{dB}}; H_{\mathrm{dB}}, K_{\mathrm{clas}}) \gg 0,$$

since the observed fringe structures are precisely what one would expect if electrons did carry a wavelength $\lambda = h/p$.

The aesthetic filter $\mathcal{A}(H_{\mathrm{dB}})$is particularly favourable:

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- Complexity $\mathcal{C}(H_{\mathrm{dB}})$is minimal: one postulate, one relation. There is no proliferation of auxiliary hypotheses; instead, existing discrete conditions are reinterpreted in a more unified vocabulary.
- Symmetry $\mathrm{Sym}(H_{\mathrm{dB}})$is high, because the hypothesis explicitly enforces a formal symmetry between radiation and matter: both are carriers of waves, both obey analogous dispersion relations. The structural role of $h$becomes homogeneous across domains.
- Correspondence $\mathcal{L}(H_{\mathrm{dB}})$is well-behaved: in the limit $h \to 0$or for macroscopic momenta p, the associated wavelength λbecomes vanishingly small, so that trajectories re-emerge as effective descriptions. Classical mechanics is thus recovered as a coarse-grained limit, preserving continuity with K_{clas}.
- Unification $\mathcal{U}(H_{\mathrm{dB}})$is substantial: optics, mechanics and the old quantum theory of orbits are drawn into a single conceptual framework governed by wave modes and phase relations.

Collecting these contributions, $\mathcal{A}(H_{\mathrm{dB}})$attains a high value: the hypothesis compresses and symmetrises the disparate indicia encoded in S_{dB}.

Once electron diffraction and related phenomena are incorporated, the fecundity term $\Phi(H_{\mathrm{dB}}; K_{\mathrm{clas}})$increases sharply. The de Broglie relation:

1. Predicts new classes of experiments (electron interferometry, diffraction by crystals, electron optics) whose success further raises W.
2. Unifies apparently distinct phenomena—orbital quantisation, interference, and the Einstein–Bohr use of $E = h\nu$—under a single guiding picture.
3. Opens a programme that leads directly to wave mechanics: Schrödinger's equations can be seen as the natural dynamical elaboration of de Broglie's kinematic insight.

In the vocabulary of our decision rule, once post-1927 data are taken into account, H_{dB}becomes a strong maximiser of

$$\exp(W(S; H, K))\, \mathcal{A}(H)\, \Phi(H; K) \text{subject to } \mathcal{L}(H) \leq \varepsilon.$$

(d) Historiographical and epistemological reading: de Broglie in the indicial lineage

From a historiographical standpoint, de Broglie's proposal has often been portrayed as a conceptual precursor to Schrödinger's wave mechanics. Within our framework, it can be read more structurally as the fourth node in a continuous indicial lineage:

- Planck reads a global spectral anomaly as a trace of discrete energy exchange.
- Einstein reads local threshold phenomena as indices of quantised radiation.
- Bohr reads structural spectral regularities as indices of quantised stationary states and transitions.
- De Broglie reads the coexistence of discrete orbits and wave phenomena as an index of a deeper wave-like substratum for matter.

What unifies these moves is not a shared ontology but a shared logic of discovery: in each case, a carefully delimited set of indicia $S$is taken as normative for the reconfiguration of the theoretical space; an abductive hypothesis $H$is introduced that (i) maximises indicial fit W, (ii) fares well under the aesthetic filter $\mathcal{A}$, (iii) promises significant fecundity Φ, and (iv) satisfies a correspondence bound $\mathcal{L}$. De Broglie's matter waves represent the point at which this logic ceases to be merely corrective of classical mechanics and begins to demand a new

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representational grammar, one where particles and waves are no longer ontological opposites but complementary aspects of a single formal structure.

In this sense, the de Broglie episode confirms that the emergence of quantum theory was not a sequence of disconnected "revolutions", but the progressive crystallisation of an indicial–abductive paradigm in which aisthesis (the sensitivity to patterns, traces and anomalies) and episteme (the disciplined articulation of hypotheses and form criteria) co-evolve. Matter waves appear, then, as a historically contingent but methodologically robust response to a zeitgeist that had already learned, with Planck, Einstein and Bohr, to treat small discrepancies and discrete structures as signs of an underlying order waiting to be deciphered.

3.5. Heisenberg (1925): spectral indices and the turn to algebraic kinematics

De Broglie's matter waves do not close the indicial–abductive itinerary; they displace it to a new level of generality. The integers in Bohr's model are no longer brute postulates but phase conditions; the coexistence of discrete spectra and wave interference ceases to be a paradox and becomes a symptom of a deeper representational duality. Yet precisely because de Broglie's proposal still moves within a largely kinematic register—assigning wavelengths to particles, reinterpreting orbits as standing waves—the tension between the old quantum theory and classical mechanics persists at the level of dynamical description: trajectories, albeit constrained, remain the tacit background. The next step in the indicial lineage therefore required a more radical reading of the traces. Instead of taking spectral regularities and selection rules as hints for a refined picture of motion in space–time, Heisenberg reads them as indices that the very kinematic vocabulary of trajectories and coordinates has become epistemically obsolete. In this sense, matrix mechanics can be understood as the moment when the indicial–abductive logic turns reflexive: the "clues" are now not only experimental anomalies and structural patterns, but also the accumulated failures and internal inconsistencies of the preceding representational schemes, which collectively point towards an algebraic re-foundation of quantum kinematics.

If Planck, Einstein, Bohr and de Broglie worked by minimally deforming the classical picture—introducing quanta into otherwise familiar dynamical frames—Heisenberg's 1925 paper signals a decisive shift: the abandonment of visualisable orbits in favour of an algebra of observables. From the standpoint of our indicial–abductive reconstruction, this is not an irrational rupture, but the next coherent response to a specific constellation of traces: unstable orbital models, ad hoc quantisation rules and spectral anomalies that jointly indicate that the classical kinematic language has ceased to be an adequate carrier of quantum information.

(a) Indicia S_H: from orbital breakdown to spectral overdetermination

By 1925, the old quantum theory presented a heterogeneous patchwork of successes and failures:

- Unstable orbital models: the Bohr–Sommerfeld extension of Bohr's model, with its multi-dimensional quantisation conditions, produced increasingly baroque orbits—precessing ellipses in higher dimensions—whose physical plausibility was doubtful and whose stability properties were opaque.
- Ad hoc quantisation rules: quantisation conditions were imposed along selected action variables without a unified rationale, often adjusted ex post to match specific spectral series.
- Spectral anomalies: the anomalous Zeeman effect, fine structure, and complex multi-electron spectra resisted systematic derivation within the Bohr–Sommerfeld

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framework and had to be treated through empirical term schemes and intricate selection rules.

We can summarise this constellation as

$$S_H \;=\; \{\text{unstable orbital models, ad hoc quantisation rules, spectral anomalies}\}.$$

The classical background K_{clas} still presupposed point particles following trajectories in space–time, described by continuous dynamical variables $q(t)$, $p(t)$satisfying Hamilton's equations. Under this background, the elements of $S_H$do not merely appear as local mismatches, but as symptoms of a deeper structural misfit: the attempt to graft discrete quantum conditions onto a continuum kinematics produces proliferating complications without a clear explanatory gain.

Within the indicial–abductive framework, this situation is itself a trace: the accumulation of incoherences and epicycles in a given representational scheme is a sign that the scheme has exhausted its capacity to naturalise the relevant patterns. In other words, the very "overfitting" of the old quantum theory becomes an index that the level at which kinematics is formulated must be changed.

(b) Heisenberg's abductive hypothesis H_H: observables as primary carriers of kinematics

Heisenberg's key move is to restrict quantum theory to quantities that are, in principle, observable—transition frequencies and amplitudes between energy levels—and to represent these quantities by arrays that encode all possible transitions between stationary states. Formally, dynamical variables such as the coordinate $q(t)$are replaced by collections of Fourier coefficients q_{mn}associated with transitions ω_{mn}between energy levels $E_m$and E_n. These coefficients are arranged in arrays

$$q \;=\; (q_{mn}),$$

and their products are defined by the composition rule

$$(qr)_{mn} \;=\; \sum_{k} \; q_{mk}\, r_{kn},$$

so that, in general,

$$qr \;\neq\; rq.$$

The abductive hypothesis $H_H$can be summarised as follows:

The correct quantum kinematics is fully captured by such arrays of transition amplitudes and their non-commutative algebra; reference to underlying classical trajectories in space–time is not only unnecessary but misleading.

This is a conjecture of high conceptual cost—since it abandons the privileged status of visualisable orbits—but it is comparatively economical in its operative core: instead of multiplying orbital models and ad hoc rules, it proposes a single algebraic structure in which frequencies, amplitudes and selection rules are encoded from the outset. In our notation, the transition

$$S_H \;\rightsquigarrow\; H_H$$

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is an abductive step that reads the failure of orbital descriptions and the success of empirical term schemes as indices that the primitive objects of the theory must be redefined: from trajectories to spectral arrays.

(c) Indicial weight $W(S_H; H_H, K_{\text{clas}})$ and aesthetic value $A(H_H)$

At the moment of Heisenberg's original proposal, the indicial weight $W(S_H; H_H, K_{\text{clas}})$ is not yet fully manifest; it requires subsequent work by Born, Jordan and Dirac to exhibit the full calculational power of the new formalism. Once matrix mechanics is consolidated, however, the penalised likelihood ratio strongly favours H_H over its classical–orbital competitors.

In the general scheme

$$W(S; H, K) \equiv \log \frac{L(S \mid H, K)}{L(S \mid H_0, K)} - \lambda \, \text{pen}(H),$$

with H_0 representing the best available Bohr–Sommerfeld-type candidate, we can say, schematically, that:

- Under H_0, the likelihood of jointly reproducing stable spectra, selection rules and anomalous Zeeman patterns without further epicyclic adjustments is low.
- Under H_H, once the matrix formalism is fully developed, the same patterns emerge naturally from the algebra of non-commuting arrays and the quantisation rule built into the commutation relations.

This is made explicit when Born and Jordan demonstrate that the new formalism reproduces the Balmer formula and other spectral regularities, and when the canonical commutation relation

$$[q, p] \equiv qp - pq = \frac{\hbar}{i}$$

(or, in Heisenberg's original normalisation, $pq - qp = \frac{h}{2\pi i}$) is derived as the quantum analogue of the classical Poisson bracket. Retrospectively, these results drive

$$W(S_H; H_H, K_{\text{clas}}) \gg 0,$$

since classical–orbital schemes render the joint success on spectra and selection rules extremely unlikely without arbitrary additions.

From the standpoint of epistemic aesthetics, the profile of $A(H_H)$ is distinctive:

- Complexity $C(H_H)$: at the level of formal machinery, complexity is high: matrix algebras, non-commutative products and operator equations. However, in terms of descriptive length, the theory is economical: a single algebraic framework replaces a host of disparate rules and special cases. Thus $C(H_H)$ is moderate once one considers compression rather than surface technicality.
- Symmetry $\text{Sym}(H_H)$: symmetry is very high. The operator formalism makes explicit use of group-theoretic structures; invariances and conservation laws are encoded in commutators and unitary transformations. The transition from classical Poisson brackets to quantum commutators preserves the structural core of canonical transformations, now in a Hilbert-space setting.

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- Coherence of limits $L(H_H)$: coherence of limits is explicitly built into the formalism: in the limit $\hbar \to 0$, commutators scale to zero and expectation values follow classical equations of motion; moreover, coarse-grained averages over many quanta recover classical trajectories as emergent patterns. Thus $L(H_H)$satisfies the correspondence constraint to a high degree.
- Unification $U(H_H)$: unification is substantial: the same algebraic apparatus applies to oscillators, rotators, hydrogen-like atoms, and, with further generalisation, to spin and multi-particle systems. Phenomena that previously required separate orbital constructions are now handled within a single operator calculus.

Collecting these contributions, the aesthetic functional

$$A(H) \;=\; e^{-\alpha C(H)}(1+\beta\,\mathrm{Sym}(H))\,e^{-\gamma L(H)}(1+\delta\,U(H))$$

takes a high value for H_H, despite the initial opacity of the formalism. The gains in symmetry and unification compensate the technical sophistication, and the carefully engineered correspondence behaviour ensures that classical mechanics is recovered as an appropriate limit.

Once one adds the fecundity component $\Phi(H_H; K_{\mathrm{clas}})$—the capacity of matrix mechanics to generate new results, support perturbation schemes and connect with later formulations such as Dirac's transformation theory—the overall decision rule

$$H^* \in \arg\max_{H\in\mathcal{H}} \exp(W(S;H,K))\,A(H)\,\Phi(H;K)\text{subject to}L(H) \le \varepsilon$$

selects $H_H$as a strong maximiser once the post-1925 body of evidence is taken into account.

(d) Historiographical and epistemological reading: Heisenberg in the indicial–abductive lineage

Within standard historiography, Heisenberg's 1925 work is often presented as an abrupt methodological revolution: a move away from the visualisable representations of classical physics towards an abstract, "symbolic" mechanics. The indicial–abductive framework proposed here allows a more graded reading. Heisenberg's step continues the same logic that guided Planck, Einstein, Bohr and de Broglie, but with a new type of trace and a new representational target:

- Planck reads a global spectral anomaly as a trace of discrete energy exchange.
- Einstein reads local threshold phenomena as indices of quantised radiation.
- Bohr reads structural regularities in line spectra as indices of quantised stationary states and transitions.
- De Broglie reads the coexistence of discrete orbits and wave phenomena as an index of an underlying wave-like substratum for matter.
- Heisenberg reads the pattern of spectral rules, anomalies and orbital breakdown as an index that the fundamental level of description is algebraic rather than geometric: the primary carriers of information are no longer trajectories but transition arrays.

What unifies these moves is the methodological backbone: in each case, a delimited set of clues S is taken as normatively binding for the reconfiguration of theoretical space; an abductive hypothesis $H$is proposed that (i) significantly raises the indicial weight Wrelative to the best available background, (ii) performs well under the aesthetic filter A, (iii) opens a fertile research programme as measured by Φ, and (iv) preserves correspondence $L$with

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entrenched regimes where they have proved reliable. Heisenberg's matrix mechanics marks the moment when this logic no longer seeks a refined picture of microscopic motion, but a new grammar of kinematics in which visualisability is subordinated to algebraic consistency with the clues.

From this perspective, the emergence of matrix mechanics appears less as an irrational break and more as a kairological crystallisation of the indicial paradigm: the point at which aisthesis—here, the trained sensitivity to the "shape" of spectral data and to the pattern of theoretical epicycles—and epistēmē—the disciplined articulation of a new operator calculus—coincide in a conjecture that reorganises the conditions of intelligibility of the micro-world. The "indices" in Heisenberg's case are thus doubly layered: they are empirical (spectra, Zeeman patterns) and meta-theoretical (the implosion of the orbital picture). The indicial–abductive logic, extended to this reflexive plane, explains why a highly non-intuitive algebra could be accepted as the core of quantum theory: it was, given the traces available and the aesthetic–epistemic norms of the time, the simplest and most coherent way to let the clues speak.

3.6. Schrödinger (1926): wave mechanics and the indicial reconstruction of quantum dynamics.

Once the spectral indices and the failure of orbital models have been read as signs that kinematics must be algebraic rather than geometric, it becomes possible to imagine that the same clue set might legitimate more than one formal realisation. It is precisely at this juncture that Schrödinger's intervention must be placed. Starting from a constellation of indicia closely related to de Broglie's matter waves and to Bohr's quantised levels, he does not reject the indicial logic that led to matrix mechanics, but performs what we might call a counter-abduction: he proposes a different representational hypothesis that preserves the quantum of action, honours the same structural clues, and satisfies analogous aesthetic criteria, but re-anchors the theory in a continuous wave picture. The transition from Heisenberg to Schrödinger thereby reveals that the indicial paradigm does not dictate a unique formalism; rather, it constrains a space of admissible conjectures within which distinct, yet empirically equivalent, grammars of the micro-world can emerge.

Schrödinger's contribution can be viewed as a counter-abduction: starting from a very similar clue set to de Broglie's and under the same pressure to account for Bohr's energy levels, he proposes a different hypothesis, wave-mechanical rather than matrix-algebraic, that turns out to be formally equivalent but aesthetically distinct. Where Heisenberg had accepted the loss of visualisability in exchange for algebraic coherence, Schrödinger attempts to recover a continuous, spacetime-based representation without sacrificing the discrete spectral structure. In doing so, he does not step outside the indicial–abductive paradigm reconstructed here; instead, he shows that a given constellation of indicia can support more than one high-scoring hypothesis in our decision rule, thereby illuminating the role of epistemic aesthetics in theory choice.

(a) Indicia S_S: de Broglie waves, standing modes and Bohr levels

Schrödinger's starting set of clues is close to de Broglie's, but read through the lens of spectral structure and eigenvalue problems:

- The de Broglie relation $\lambda = h/p$, associating a wavelength $\lambda$to a particle of momentum p, suggests a wave-like kinematics for matter.
- Standing waves and quantised normal modes in classical systems (vibrating strings, cavities) provide a familiar mechanism by which discrete spectra emerge from continuous wave equations via boundary conditions.

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- The success of Bohr's energy levels for hydrogen and hydrogen-like systems indicates that any admissible dynamics must reproduce those discrete values as eigenvalues or mode numbers.

We can summarise this constellation as

$S_S = \{\lambda = h/p$, standing waves and quantised modes, success of Bohr's energy levels$\}$.

Here the background $K$is an enriched classical field picture: wave equations, eigenvalue problems, normal modes and variational principles constitute the natural vocabulary for dynamics. Under this background, the clues in $S_S$point almost irresistibly towards a field-like description of matter in which quantisation appears as a boundary-value property rather than as a bare postulate. In indicial terms, the conjunction of de Broglie's wavelength, Bohr's discrete levels and the well-known behaviour of standing waves functions as a cluster of traces suggesting that the appropriate representational space is that of wave fields on configuration space.

(b) Hypothesis $H_S$and its structure: a wave equation for matter

Schrödinger's hypothesis is that the dynamics of a non-relativistic particle of mass $m$in a potential $V$is governed by a wave equation for a complex function ψdefined on configuration space. For stationary states he writes an eigenvalue problem

$$\hat{H}\,\psi = E\,\psi, \hat{H} = -\frac{\hbar^2}{2m}\,\nabla^2 + V,$$

and for time evolution the equation

$$\mathrm{i}\hbar\,\frac{\partial\psi}{\partial t} = \hat{H}\,\psi.$$

This represents an abductive move: if de Broglie's matter waves exist and if quantisation corresponds, in classical wave systems, to boundary conditions on standing waves, then a wave equation whose eigenvalues reproduce Bohr's energy levels provides a natural explanation of the spectra. The hypothesis $H_S$can be summarised as follows:

There exists a complex wave field ψon configuration space whose dynamics is governed by a linear differential operator $\hat{H}$such that the allowed energy levels of bound systems are the eigenvalues of $\hat{H}$, and transition frequencies correspond to differences between these eigenvalues.

In our framework, $H_S$is a low-complexity conjecture: it introduces a single new dynamical object (the wave function) and a single structural postulate (linear unitary evolution under $\hat{H}$), both formulated in terms of operators and differential equations already familiar from classical wave theory. The discrete energies of Bohr's model are no longer imposed by fiat; they emerge as spectral properties of $\hat{H}$under appropriate boundary conditions.

From the standpoint of the indicial logic, Schrödinger's abductive step can therefore be written schematically as

$S_S \dashrightarrow H_S$:"interpret Bohr's levels as eigenvalues of a matter-wave Hamiltonian."

(c) Indicial weight $W(S_S; H_S, K)$, aesthetic value $A(H_S)$and programme fecundity $\Phi(H_S; K)$

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The indicial weight $W(S_S; H_S, K)$becomes very high once Schrödinger shows that his equation reproduces hydrogen-like levels, fine structure corrections (with relativistic refinements), and other results of the old quantum theory. In the penalised likelihood scheme outlined in §2, the ratio

$$W(S_S; H_S, K) = \log \frac{L(S_S \mid H_S, K)}{L(S_S \mid H_0, K)} - \lambda \,\mathrm{pen}(H_S)$$

is strongly positive when H_0 is taken as the best Bohr–Sommerfeld-type competitor. Under H_0, the likelihood of obtaining discrete spectra, correct degeneracies and selection rules without additional epicyclic assumptions remains modest; under H_S, the same features emerge in a unified way from the spectral theory of $\widehat{H}$.

The aesthetic profile $A(H_S)$is particularly favourable when decomposed into its components:

- Complexity C: at the formal level, the machinery of partial differential equations and Hilbert spaces is non-trivial, but in terms of descriptive length the theory is compact: a single linear operator encodes bound states, scattering, tunnelling and interference phenomena. There is no proliferation of special-purpose quantisation rules. Thus $C(H_S)$is moderate once compression is taken as the relevant measure.

- Symmetry Sym: the Hamiltonian operator $\widehat{H}$encodes spatial symmetries and conservation laws in a transparent way: rotational invariance, translational symmetry and other group-theoretic properties appear directly in the form of $\widehat{H}$and in the structure of its eigenfunctions. Noether-type correspondences between symmetries and conserved quantities carry over naturally to the wave-mechanical setting. Hence $\mathrm{Sym}(H_S)$is high.

- Coherence of limits L: coherence of limits is well behaved. In the WKB regime and for large quantum numbers, Schrödinger's equation yields approximate classical trajectories as the flow lines of wave-packet centres, and action variables reappear in the semiclassical phase. In the limit $\hbar \to 0$, the phase of ψsatisfies a Hamilton–Jacobi equation and the classical equations of motion are recovered as characteristic curves. Thus $L(H_S)$satisfies the correspondence constraint to a high degree.

- Unification U: unification is substantial: the same equation treats bound states, scattering, tunnelling and interference within a single framework. Phenomena that had previously required separate constructions (e.g. Bohr orbits, Sommerfeld ellipses, ad hoc quantisation in multiply periodic systems) are subsumed under a common differential operator and boundary-value theory. Consequently, $U(H_S)$is high.

Putting these components together, the aesthetic functional

$$A(H_S) = \mathrm{e}^{-\alpha C(H_S)}\,(1 + \beta\,\mathrm{Sym}(H_S))\,\mathrm{e}^{-\gamma L(H_S)}\,(1 + \delta\,U(H_S))$$

takes a large value: the gains in symmetry, unification and limit coherence more than compensate the formal sophistication of the mathematical apparatus.

Once we add the fecundity term $\Phi(H_S; K)$, the score of $H_S$increases further. The Schrödinger equation:

- Predicts novel phenomena such as quantum tunnelling and quantised scattering cross-sections, which were not built into the old quantum theory;

- Unifies diffraction, bound-state spectra and dispersion in a single wave-mechanical picture;

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- Technifies the theory by providing powerful calculational tools (perturbation theory, variational methods, eigenfunction expansions) that can be systematically extended.

Within the global decision rule

$$H^* \in \arg\max_{H \in \mathcal{H}} \exp(W(S; H, K))\, A(H)\, \Phi(H; K) \text{subject to } L(H) \leq \varepsilon,$$

Schrödinger's $H_S$emerges, after 1926, as a rival maximiser to Heisenberg's H_H. Both hypotheses yield high W, high $A$and high Φunder closely related clue sets S; the subsequent proof of formal equivalence between matrix mechanics and wave mechanics can thus be read as a paradigmatic case of multi-realisation of a single indicial–abductive core.

(d) Historiographical and epistemological reading: Schrödinger in the indicial–abductive lineage

From a standard historiographical perspective, Schrödinger's wave mechanics is often portrayed as a "softer" alternative to Heisenberg's abstract symbolism: a return to visualisability, continuity and differential equations. In the indicial–abductive reconstruction proposed here, this contrast is preserved but reinterpreted. Schrödinger does not reverse the quantum turn inaugurated by Planck, Einstein, Bohr, de Broglie and Heisenberg; he replays it at the level of representation.

- Planck reads a global spectral anomaly as a trace of discrete energy exchange.
- Einstein reads local threshold phenomena as indices of quantised radiation.
- Bohr reads structural regularities in line spectra as indices of quantised stationary states and transitions.
- De Broglie reads the coexistence of discrete orbits and wave phenomena as an index of an underlying wave-like substratum for matter.
- Heisenberg reads the pattern of spectral rules, anomalies and orbital breakdown as an index that the fundamental level of description is algebraic rather than geometric.
- Schrödinger, finally, reads de Broglie's matter waves and Bohr's levels as indices that a continuous wave equation on configuration space can encode the same discrete structure, provided the Hamiltonian operator is appropriately chosen.

What unifies these moves is not a shared metaphysics of quanta, but a shared method: in each case, a delimited set of clues $S$is treated as normatively binding; an abductive hypothesis $H$is introduced that significantly raises the indicial weight Wrelative to the best available background, fares well under the aesthetic filter A, opens a fertile research programme as measured by Φ, and preserves correspondence $L$with entrenched regimes. Schrödinger's contribution shows that, once the indicial–abductive paradigm is in place, it can underwrite representational pluralism: different formalisms can qualify as equally legitimate carriers of the same quantum clues.

In this sense, wave mechanics appears not as a retreat from the algebraic boldness of matrix mechanics, but as an alternative kairological crystallisation of the same underlying logic of discovery. Aisthesis and epistēmē co-evolve once more: the trained sensitivity to the "shape" of spectra and to the behaviour of standing waves (aisthesis) encounters the mathematical resources of spectral theory and variational calculus (epistēmē) and condenses into a conjecture—H_S—that reorganises the conditions of intelligibility of quantum phenomena. The fact that two such conjectures, $H_H$and H_S, can coexist, compete and finally be shown formally equivalent, is itself an instructive trace: an index that the

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indicial–abductive paradigm does not fix a unique image of the micro-world, but a family of admissible grammars constrained by the same economy of clues, evidence and form.

3.7. Pauli (1925–1930): exclusion, spin and the indicial completion of quantum statistics

Once both matrix mechanics and wave mechanics are in place, the indicial–abductive trajectory of early quantum theory undergoes a further displacement. The basic kinematics has been doubled—algebraic and wave-theoretic formalisms coexist as empirically equivalent grammars—but the clue space $S$is not exhausted. New, stubbornly systematic traces come into focus at a different scale: the fine structure of the periodic table, shell saturation, anomalous Zeeman patterns and the apparent violations of conservation laws in beta decay. These are not indices of single-particle dynamics, but of how many-particle systems organise themselves under quantum constraints. In this sense, the next step in the lineage does not consist in yet another representational reform of kinematics, but in the abductive introduction of structural principles and new entities that complete the statistical and symmetry profile of the theory. It is here that Pauli's work must be situated: as the moment when the indicial–abductive logic is extended from the level of individual trajectories or waves to that of admissible quantum states and their occupation, and from spectral anomalies to the global bookkeeping of charge, spin and energy in nuclear processes.

Pauli's contributions exemplify how the indicial–abductive paradigm extends to statistical and structural principles that govern ensembles of quantum systems rather than individual particles. Where Planck, Einstein, Bohr, de Broglie, Heisenberg and Schrödinger had progressively reconfigured the kinematics and dynamics of single electrons and photons, Pauli operates at the level of permissible configurations and hidden degrees of freedom, taking as clues the regularities of the periodic table, the anomalies of the Zeeman effect and the saturation properties of atomic shells. His exclusion principle and neutrino hypothesis are not deductions from a prior dynamics; they are abductive responses to a constellation of traces that resist accommodation within the existing formalism, yet display a high degree of structural coherence.

(a) Indicia S_P: periodic structure, Zeeman anomalies and saturation

The key traces to which Pauli responds can be organised into a characteristic clue set S_P:

- Periodic table regularities: the empirical organisation of elements into periods and groups, with regular patterns in valence and shell filling, suggests a discrete internal structure of atomic states, with strict occupancy limits in each shell.
- Anomalous Zeeman splitting: spectroscopic observations of line splitting in magnetic fields reveal a two-valuedness that simple orbital angular momentum cannot explain, indicating an additional binary degree of freedom attached to the electron.
- Electron saturation and stability: atoms exhibit sharp saturation: once a shell is "full", additional electrons occupy higher shells rather than crowding the same energy level, and the chemical stability of noble gases points to closed-shell configurations.

We may summarise these clues as

$$S_P = \{\text{periodic table regularities, Zeeman anomalies, electron saturation}\}.$$

Within the then-available background K—Bohr–Sommerfeld orbits enriched by spin-orbit couplings, matrix and wave mechanics for one-electron systems—these indicia are difficult to naturalise. Orbital models can be tuned to reproduce some features, but the saturation pattern and the two-valuedness in magnetic behaviour resist simple geometrical

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interpretation. In the indicial language of this paper, S_P constitutes a structured set of traces pointing not to a new kind of trajectory or wave equation, but to constraints on how quantum states can be occupied and to a hidden internal degree of freedom associated with the electron.

(b) Hypotheses H_{excl} and H_ν: exclusion and the neutrino as abductive completions

Pauli's first major abductive step is the exclusion principle, which we denote by H_{excl}. In its mature form, it can be stated as:

No two electrons in an atom can occupy the same quantum state.

Formally, if a one-electron state is labelled by quantum numbers (n, ℓ, m, s), with s encoding a two-valued degree of freedom, then no pair (i, j) may satisfy

$$(n_i, \ell_i, m_i, s_i) = (n_j, \ell_j, m_j, s_j), i \neq j.$$

This principle is not derived from the dynamics of the Schrödinger equation; it is introduced to render natural the observed regularities in atomic structure. The periodic table, shell filling and saturation are thus re-described as indices of a deeper combinatorial constraint on state occupancy.

Pauli's second abductive move, which we denote H_ν, addresses a different but related clue set: the continuous energy spectrum of beta decay and the apparent non-conservation of energy and angular momentum at the nuclear level. Faced with these anomalies, Pauli proposes the existence of a new light, neutral particle—the neutrino—such that the missing energy and angular momentum are carried away by this unobserved entity. The hypothesis can be summarised as:

There exists a neutral fermion with very small mass that participates in beta decay, ensuring exact conservation of energy, momentum and angular momentum in each event.

Both H_{excl} and H_ν share the same indicial–abductive structure: they are compact principles or entity-postulates introduced because they drastically improve the fit between a recalcitrant clue set S and the global conservation and symmetry structure encoded in K.

(c) Evaluation via $W(S_P; H, K)$, aesthetic value $A(H)$ and programme fecundity $\Phi(H; K)$

Within the general scheme of §2, the indicial weight of the exclusion principle can be written as

$$W(S_P; H_{\text{excl}}, K) = \log \frac{L(S_P \mid H_{\text{excl}}, K)}{L(S_P \mid H_0, K)} - \lambda \, \text{pen}(H_{\text{excl}}),$$

where H_0 denotes the best available attempt to account for periodicity and saturation using only orbital quantum numbers and one-electron Schrödinger theory.

Under H_0, the likelihood of reproducing the fine-grained structure of the periodic table, including shell capacities and the stability of noble gases, is modest; it requires intricate and ad hoc assumptions about electron repulsion and orbital configurations. Under H_{excl}, the same phenomena follow in a simple and uniform way from a single occupancy rule combined with the quantum numbers delivered by the one-electron theory. Hence $W(S_P; H_{\text{excl}}, K) \gg 0$.

A similar reasoning applies to the neutrino hypothesis H_ν once beta spectra are treated as reliable indicia of missing energy and angular momentum: classical "two-body" beta-decay models render the observed continuous spectrum extremely unlikely, whereas the introduction of a third light particle restores a natural fit between data and conservation

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laws. Retrospectively, after experimental confirmation, we can also assign $W(S_\beta; H_\nu, K) \gg 0$for the relevant clue set S_β.

The aesthetic filter $A(H)$is again favourable for both hypotheses:

- Complexity C: for H_{excl}, complexity is minimal: a single, sharp principle stipulates a constraint on state occupancy. For H_ν, the postulation of one new particle with well-defined properties is also a low-complexity move compared to the proliferation of ad hoc violations of conservation laws. Thus $C(H_{\text{excl}})$and $C(H_\nu)$are both low.
- Symmetry Sym**:** the exclusion principle is closely tied, in later developments, to the antisymmetry of fermionic wave functions and to the representation theory of the permutation group; it enforces a deep symmetry–statistics connection (spin–statistics theorem) that unifies spectroscopic and statistical behaviour. The neutrino hypothesis preserves global conservation symmetries (energy, momentum, angular momentum, lepton number) at the microscopic level. In both cases, $\text{Sym}(H)$is high.
- Coherence of limits L: at macroscopic scales, both principles are compatible with established thermodynamics and chemistry: exclusion underlies the stability of ordinary matter and the Pauli pressure in dense systems, while neutrino emission remains negligible in everyday processes. Classical behaviour is recovered as an effective limit of large ensembles. Thus the coherence-of-limits component $L(H)$is well within the correspondence bound.
- Unification U: exclusion unifies disparate domains: spectroscopic regularities, chemical valence, the stability of matter and, eventually, the structure of degenerate stars. The neutrino unifies beta decay with the otherwise well-confirmed conservation principles of classical and quantum physics. Consequently, $U(H_{\text{excl}})$and $U(H_\nu)$are substantial.

Collecting these contributions, the aesthetic functional

$$A(H) = \mathrm{e}^{-\alpha C(H)}(1 + \beta\,\text{Sym}(H))\,\mathrm{e}^{-\gamma L(H)}(1 + \delta\,U(H))$$

takes a high value for both $H_{\text{excl}}$and H_ν. They are formally simple, symmetry-enhancing, correspondence-respecting and strongly unifying.

The programme fecundity $\Phi(H; K)$further strengthens their status. Exclusion leads directly to:

- Fermi–Dirac statistics and the distinction between fermions and bosons;
- Explanations of the stability and compressibility of matter, including white-dwarf and neutron-star structure;
- A coherent account of shell structure and periodic trends in chemistry.

The neutrino hypothesis, in turn, opens a research programme in weak interactions, nuclear structure and particle physics, culminating in the experimental detection of neutrinos and the elaboration of the Standard Model. In both cases, $\Phi(H; K)$is very high.

Within the global decision rule

$$H^* \in \arg\max_{H\in\mathcal{H}} \exp(W(S; H, K))\,A(H)\,\Phi(H; K)\text{subject to } L(H) \le \varepsilon,$$

Pauli's $H_{\text{excl}}$and $H_\nu$score extremely well despite their initially conjectural character. They illustrate that the indicial–abductive paradigm is not restricted to "mechanistic" models of

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motion; it also governs the introduction of structural principles and entities justified by their capacity to reconcile local anomalies with global constraints.

(d) Historiographical and epistemological reading: Pauli in the indicial–abductive lineage

Placed within the lineage reconstructed in this paper, Pauli appears as the operator who shifts the indicial–abductive logic from kinematics and dynamics to statistics and symmetry:

- Planck reads a global spectral anomaly as a trace of discrete energy exchange.
- Einstein reads local threshold phenomena as indices of quantised radiation.
- Bohr reads structural regularities in line spectra as indices of quantised stationary states and transitions.
- De Broglie reads the coexistence of discrete orbits and wave phenomena as an index of an underlying wave-like substratum for matter.
- Heisenberg reads the pattern of spectral rules, anomalies and orbital breakdown as an index that the fundamental level of description is algebraic rather than geometric.
- Schrödinger reads de Broglie's matter waves and Bohr's levels as indices that a continuous wave equation on configuration space can encode the same discrete structure.
- Pauli, finally, reads the periodic table, Zeeman anomalies, saturation and beta spectra as indices that quantum states obey a non-classical combinatorics and that unseen carriers of energy and angular momentum may be required to preserve global symmetry constraints.

The continuity is methodological rather than purely doctrinal. In each case, a carefully delimited set of indicia $S$is taken as normatively binding; an abductive hypothesis $H$is introduced that:

(i) significantly raises the indicial weight Wrelative to the best available background;

(ii) performs well under the aesthetic filter A, balancing simplicity, symmetry, correspondence and unification;

(iii) opens a fertile research programme as measured by Φ; and

(iv) respects a correspondence constraint $L$with entrenched regimes.

Pauli's exclusion principle and neutrino hypothesis show that this logic naturally extends to the statistical and structural layer of the theory. The emerging quantum paradigm is thus completed not only by new equations of motion, but by new constraints on what counts as an admissible state and by new carriers of conserved quantities.

From a philosophical point of view, Pauli's episode also displays a characteristic feature of the indicial–abductive paradigm in its mature form: its willingness to let "negative" information—missing states, forbidden configurations, absent energy—function as a clue. The absence of certain spectral lines, the impossibility of overfilling shells, the apparent deficit in beta decay are all treated as traces of an order that is not yet explicit in the formalism. In that sense, Pauli's work makes visible a further dimension of the quantum zeitgeist: the idea that the structure of what does *not* occur can be as normatively revealing as the structure of what does, and that aisthesis and epistēmē must extend to the combinatorics of absence if the theory is to become statistically and symmetrically complete.

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4. Conclusion: Towards a General Theory of Indicial Logic in scientific discovery

The reconstruction undertaken in this article suggests that the formative decades of quantum theory (1900–1930) can be read, at micro-historical scale, as a sequence of epistemic acts governed not by revolutionary discontinuities but by an underlying indicial logic. In each episode—Planck's discretised oscillators, Einstein's light quanta, Bohr's stationary states, de Broglie's matter waves, Heisenberg's algebraic kinematics, Schrödinger's wave mechanics, Pauli's structural principles—the decisive move consists in reading a delimited set of traces (spectral curvatures, thresholds, arithmetic regularities, missing quantities, forbidden configurations) as signs of latent structure, and in articulating an abductive conjecture that maximises explanatory and formal economy under the constraints of an inherited background. What emerges, therefore, is not merely a new physical theory but a generalisable method of discovery, in which *aisthesis* and *epistēmē*—the fine-grained perception of phenomenological detail and the disciplined elaboration of theoretical form—operate in synchrony.

4.1. The micro-historical grammar of abductive reasoning

The historically situated episodes examined here confirm that the most productive cognitive unit in the quantum germ is not the broad theoretical framework nor the large-scale experiment, but the micro-historical clue: a resistant measurement, a persistent mismatch, an anomalous regularity. Following Ginzburg's indiciary paradigm, each episode demonstrates how seemingly marginal discrepancies—ultraviolet divergence, threshold frequencies, Balmer–Ritz arithmetic, anomalous Zeeman splittings, missing beta energy—acquire normative power disproportionate to their empirical modesty. They act as kairological signals: occasions whose interpretive potential becomes actual only when the appropriate triadic synchrony (trace–background–interpretant) is reached.

This micro-historical emphasis revises the conventional historiography of quantum theory. Rather than interpreting the period as a sequence of radical ruptures or paradigm shifts, the indicial reconstruction shows that physicists repeatedly engaged in a logic of disciplined attention: a practice of interpreting the smallest recalcitrant details as windows into hidden structure. The clue is not noise to be filtered but *information* to be amplified. In turn, the hypothesis is not a metaphysical leap but a minimal conjecture designed to render those clues *natural* within a reconfigured space of representations.

4.2. Aisthesis–epistēmē coupling as a condition of discovery

Across all quantum episodes, discovery arises not from algorithmic derivation nor from aesthetic intuition alone, but from a coupling between aisthesis and epistēmē. Aisthesis provides the trained susceptibility to the "shape" of experimental and theoretical mismatches—spectral tails, degeneracies, selection rules, combinatorial absences. Epistēmē supplies the formal resources to transform this susceptibility into an explicit hypothesis: a restriction on energy exchange (Planck), a linear frequency–energy law (Einstein), a stationary-state architecture (Bohr), a wave-number assignment (de Broglie), a non-commutative algebra (Heisenberg), a wave equation (Schrödinger), a combinatorial constraint or a neutral particle (Pauli).

This synergy displaces the Platonic hierarchy in which aisthesis is epistemically inferior. In the quantum germ, aisthesis is the operative threshold of reason: the workshop in which traces become signs. And epistēmē is not a deductive machinery but a normative filter, selecting among conjectures those that satisfy criteria of parsimony, symmetry, limit coherence and unification. This is why the aesthetic functional introduced in §2 is not ornamental but *epistemically binding*: it articulates form criteria tacitly operative in historical decisions of theory choice.

4.3. The structure of indicial logic: a general model

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The model developed in §§1–2 now appears not as a retrospective imposition but as a formal distillation of the micro-historical practice exhibited in §§3. Its components—indicial weight , aesthetic filter , programme fecundity and correspondence constraint — map directly onto historical behaviour:

- Indicial weight captures the fact that each quantum hypothesis begins as a response to a circumscribed, high-salience anomaly.
- Aesthetic value explains why communities converged on conjectures that were not only empirically viable but also *structurally economical*.
- Fecundity accounts for the preference for hypotheses opening new domains of phenomena, calculational techniques or conceptual unifications.
- Correspondence codifies the insistence—visible in Planck, Bohr, Heisenberg and Schrödinger—that new theories must recover classical behaviour in the appropriate limits.

The global decision rule

formalises the ampliative procedure underlying the early quantum discoveries: models are chosen not because they maximise truth in an absolute sense, but because they optimise a multi-criteria functional under conditions of weak, heterogeneous evidence.

4.4. Methodological consequences

The reconstruction yields several broader methodological consequences:

1. Weak signals can be normatively decisive: the quantum germ shows that theory formation often begins not with robust regularities but with small, stubborn irregularities that classical backgrounds cannot naturalise.

2. Abduction is not pre-scientific intuition but a disciplined inferential rule: when formalised by indicial weight and aesthetic constraints, abduction becomes a *testable*, *auditable* component of discovery.

3. Aesthetic criteria are epistemically operative: simplicity, symmetry, limit coherence and unification acquire normative force not because they are beautiful, but because they structure the search space under uncertainty.

4. Scientific rationality is non-monotonic: new clues can overturn earlier conjectures, and representational pluralism (matrix vs wave mechanics) is admissible when multiple hypotheses score similarly under the decision rule.

5. Micro-historical reasoning scales up: what begins as a forensic reading of anomalies becomes, through successive abductive consolidations, a large-scale transformation of the theoretical landscape.

4.5. The quantum germ as proof of concept

From a philosophical standpoint, the quantum germ functions as a proof-of-concept for the indicial paradigm. It shows that:

- revolutionary theories may arise from *minimal yet normatively loaded conjectures*;
- epistemic progress can be reconstructed without invoking incommensurable paradigms;
- scientific communities employ tacit but stable criteria of form that can be formalised;
- representational pluralism is compatible with shared evidential and aesthetic norms.

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Most importantly, the quantum germ illustrates that the growth of knowledge proceeds through a delicate balance between fidelity to traces (indicia) and creativity in conjecture: a form of rationality that neither reduces discovery to method nor romanticises it as inspiration.

4.6. Future Directions

The indicial–abductive model developed here invites several lines of further research:

- Comparative micro-histories of other scientific revolutions (thermodynamics, genetics, plate tectonics) analysed through the same decision rule.
- Quantitative calibration of the aesthetic hyperparameters across historical periods.
- Model-selection frameworks that integrate indicial logic with Bayesian and information-theoretic epistemology.
- Digital micro-historiography, using data-mining to track the emergence, persistence and decay of clue sets in laboratory notebooks and correspondence.
- Pedagogical applications: teaching discovery not as a linear method but as a disciplined art of reading traces.

The overarching lesson of this study is that the early quantum theorists—far from being isolated geniuses or architects of sudden ruptures—were practitioners of a shared, though tacit, logic of clues. Their discoveries were not miracles but *kairological crystallisations*: moments in which aisthesis and epistēmē align such that a small trace becomes the seed of a new grammar of nature. The indicial paradigm provides the conceptual and formal tools to make this process auditable, historically situated and philosophically intelligible. In doing so, it restores to the history of science the subtlety it deserves: a domain where attention to the smallest detail can reconfigure the largest theoretical architectures.

References

Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control, 19*(6), 716–723. https://doi.org/10.1109/TAC.1974.1100705

Aliseda, A. (2006). *Abductive reasoning: Logical investigations into discovery and explanation*. Springer. https://doi.org/10.1007/1-4020-3907-7

Bohr, N. (1913). On the constitution of atoms and molecules. *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 26*(151), 1–25. https://doi.org/10.1080/14786441308634955

Daston, L., & Galison, P. (2007). *Objectivity*. Zone Books. https://www.zonebooks.org/books/5-objectivity

Davisson, C. J., & Germer, L. H. (1927). Diffraction of electrons by a crystal of nickel. *Physical Review, 30*(6), 705–740. https://doi.org/10.1103/PhysRev.30.705

de Broglie, L. (1924). *Recherches sur la théorie des Quanta.* Physique [physics]. Migration- université en cours d'affectation, 1924. Français. NNT. tel-00006807. https://theses.hal.science/tel-00006807v1

Einstein, A. (1905). On a heuristic point of view concerning the production and transformation of light. *Annalen der Physik, 17*, 132–148. (Reprinted in *The collected papers of Albert Einstein, Vol. 2.*) https://doi.org/10.1002/andp.19053220607

Ginzburg, C. (1992). *Clues, myths, and the historical method* (J. Tedeschi & A. C. Tedeschi, Trans.). Johns Hopkins University Press. (Original essays published in Italian in 1979.) https://archive.org/details/cluesmythshistor0000ginz/page/n1/mode/2up

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Hacking, I. (2012). *Representing and intervening: Introductory topics in the philosophy of natural science*. Cambridge University Press. https://doi.org/10.1017/CBO9780511814563

Heisenberg, W. (1925). Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen. *Zeitschrift für Physik, 33*(1), 879–893. https://doi.org/10.1007/BF01328377

Ivanova, M., & French, S. (Eds.). (2020). *The aesthetics of science: Beauty, imagination and understanding*. Routledge. https://doi.org/10.4324/9780429030284

Jaynes, E. T. (2003). *Probability theory: The logic of science*. Cambridge University Press. https://assets.cambridge.org/052159/2712/sample/0521592712WS.pdf

Koyré, A. (1968). *From the closed world to the infinite universe*. Johns Hopkins University Press. https://press.jhu.edu/books/title/1632/closed-world-infinite-universe

Kuhn, T. S. (1978). *Black-body theory and the quantum discontinuity, 1894–1912*. Oxford University Press. https://archive.org/details/blackbodytheoryq0000kuhn/page/n9/mode/2up

Lenard, P. (1902). Über die lichtelektrische Wirkung. *Annalen der Physik, 8*(5), 149–198. https://doi.org/10.1002/andp.19023130510

MacKay, D. J. C. (2003). *Information theory, inference, and learning algorithms*. Cambridge University Press. https://www.cambridge.org/gb/universitypress/subjects/computer-science/pattern-recognition-and-machine-learning/information-theory-inference-and-learning-algorithms#resources

McAllister, J. W. (1996). *Beauty and revolution in science*. Cornell University Press. https://www.sortie-nd.org/lme/Bayesian%20methods%20in%20ecology/Royall_2004_Likelihood_Paradigm.pdf?utm_source

Noether, E. (1971). Invariant variation problems (M. A. Tavel, Trans.). *Transport Theory and Statistical Physics, 1*(3), 183–207. https://doi.org/10.1080/00411457108231446 (Original work published 1918.)

Pais, A. (2005). *"Subtle is the Lord…": The science and the life of Albert Einstein*. Oxford University Press. https://doi.org/10.1093/oso/9780192806727.001.0001

Pauli, W. (1925). Über den Zusammenhang des Abschlusses der Elektronengruppen im Atom mit der Komplexstruktur der Spektren. *Zeitschrift für Physik, 31*(1), 765–783. https://doi.org/10.1007/BF02980631

Pauli, W. (1930/2018). Pauli's neutrino letter (1930). In CERN Library (Ed.), *December 1930 – Pauli's neutrino letter* (online exhibit). Retrieved from https://library.cern/archives/history_CERN/historical_images/month-88-years-ago?utm_source

Peirce, C. S. (1998). *The essential Peirce: Selected philosophical writings, Volume 2 (1893–1913)* (N. Houser, C. Kloesel, & The Peirce Edition Project, Eds.). Indiana University Press. https://archive.org/details/essentialpeirces0000peir

Planck, M. (1901). On the law of distribution of energy in the normal spectrum. *Annalen der Physik, 4*(553), 1–11. https://doi.org/10.1002/andp.19013090310 (English translation in D. ter Haar (Ed.), *The old quantum theory*, Pergamon, 1967.)

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Ramírez Suárez, A. (2007). Charles Sanders Pierce: semeiosis y ocasión oportuna. *Diálogos*: (90), 261-294. https://revistas.upr.edu/images/dialogos/2007/n90/a16.pdf

Rissanen, J. (1978). Modeling by shortest data description. *Automatica, 14*(5), 465–471. https://doi.org/10.1016/0005-1098(78)90005-5

Rojas Parma, L. (2015). Protágoras y el significado de aisthesis. *Revista de Filosofía*, 71, 127–149. http://dx.doi.org/10.4067/S0718-43602015000100011

Royall, R. (1997). *Statistical evidence: A likelihood paradigm*. Chapman & Hall/CRC. https://archive.org/details/statisticalevide0000roya/page/n9/mode/2up

Schrödinger, E. (1926). Quantisierung als Eigenwertproblem (Parts I–IV). *Annalen der Physik, 79–80*. (English translation in E. Schrödinger, *Collected papers on wave mechanics*, AMS Chelsea, 1982.) https://doi.org/10.1002/andp.19263840602

Schwarz, G. (1978). Estimating the dimension of a model. *The Annals of Statistics, 6*(2), 461–464. https://doi.org/10.1214/aos/1176344136

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